A Fission Matrix Approach To Calculate Pin-wise 3D Fission Density Distribution

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Outline

- Introduction
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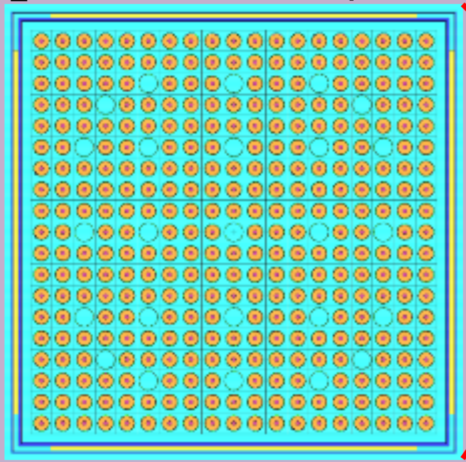
- Fission Matrix (FM) Method
- Calculation of FM Coefficients
- Results
  - Eigenvalue calculations
  - Fixed-source subcritical multiplication
  - Fresh and spent fuel cases
- Conclusions

Introduction

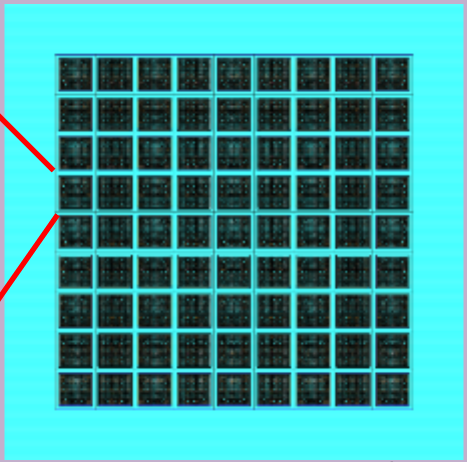
- Spent fuel pool neutronics calculations are important
  - Criticality safety
  - Safeguards and verification
- Full Monte Carlo calculations face difficulties in this area
  - Convergence is difficult to low coupling between areas
  - Computation times are very long
  - Changing pool configuration requires complete recalculation
- Fission Matrix method can address some of these issues
  - Fission matrix coefficients are pre-calculated using Monte Carlo
  - Computation times are much shorter, with no convergence issues
  - Detailed fission distributions are available at the pin level
  - Changing pool assembly configuration does not require new pre-calculations

Reference Spent Fuel Pool

- Being developed for I2S-LWR* reactor design
- 19x19 U_3Si_2 fuel assemblies
- 4.45 and 4.95 w/o U-235
- Metamic® absorbers (Al-B4C) used between assemblies
- Burnup up to 55 GWd/MTU



One 19x19 Assembly



9x9 segment of spent fuel pool

Whole pool made up of 8 9x9 segments

*Integral Inherently Safe – LWR, funded by DOE, project led by Georgia Tech

Types of Spent Fuel Pool Calculation

- Criticality calculation
  - Determine the eigenvalue (k) to ensure subcriticality
  - Loosely coupled system – very slow convergence with Monte Carlo, possibly false convergence
- Subcritical fixed source calculation
  - Neutron source from spontaneous fission and (alpha,n) interactions
  - Gives the actual neutron flux in the system for detector response, etc
  - Monte Carlo has high uncertainties for local variables (e.g., detector response, pin-wise fission rate)
  - Use the subcritical multiplication M as a metric

$M \neq \frac{1}{1-k}$ unless S is equal to the fundamental eigenfunction

Fission Matrix Method

- Fission in each cell i due to contributions from all other cells j

$$F_i = \sum_{j=1}^{N} (a_{i,j} F_j + b_{i,j} S_j)$$

  - F_i is fission source, S_i is fixed source in cell i
  - $a_{i,j}$ is the number of fission neutrons produced in cell i due to a fission neutron born in cell j. $b_{i,j}$ is the same but for independent source neutrons
- Written in matrix form:

$$\vec{F} = A\vec{F} + \mathrm{B}\vec{S}$$

- Subcritical multiplication defined:

$$M = \frac{F + S}{S}$$

- In source-free eigenvalue form:

$$\vec{F} = \frac{1}{k} A\vec{F}$$

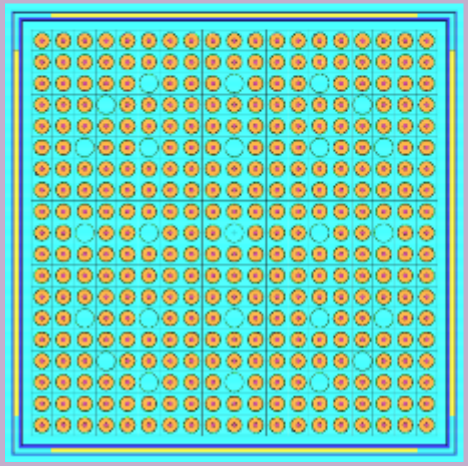
- Chief difficulty: How to determine A? And what computational size to use (i.e., N)?

Calculation of Fission Matrix Coefficients

- Run fixed source calculation for each cell j
- Tally total fission neutrons produced in every other cell i
- The coefficients can then be calculated

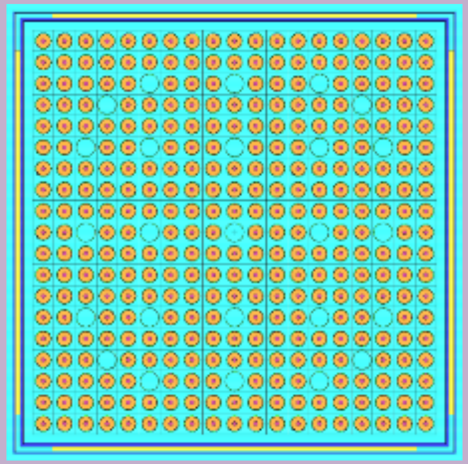
$$a_{i,j} = \frac{\phi_i \nu \sigma_f}{S_j}$$

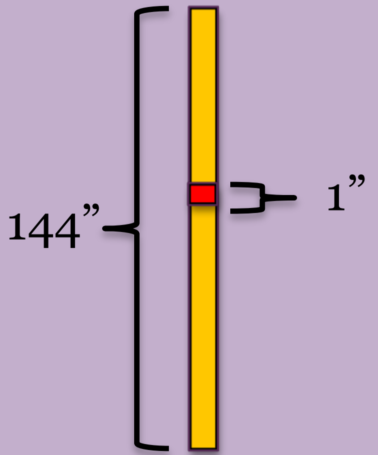
- Requires knowledge of the within-cell source distribution (spatial and energy)



Fission Matrix Cell Size

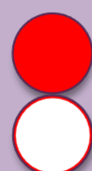
- Cell size affects speed and accuracy
- As the computational cell size, use one inch axial slice of single pin
  - 81 assemblies in a single 9x9 section of the pool
  - 336 pins per assembly
  - 144 axial levels per pin
  - $N = 9 * 9 * 336 * 144 = 3{,}919{,}104$ total fission matrix cells
- Standard FM would require 3,919,104 separate fixed-source calculations to determine the A matrix
  - One calculation for each slice of each pin
  - A matrix of size $N * N = 1.5 * 10^{13}$ total coefficients



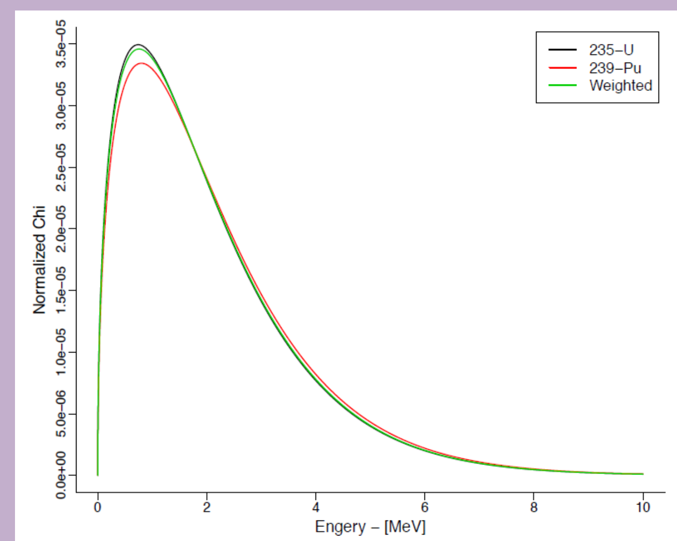


Within-cell Source Distribution

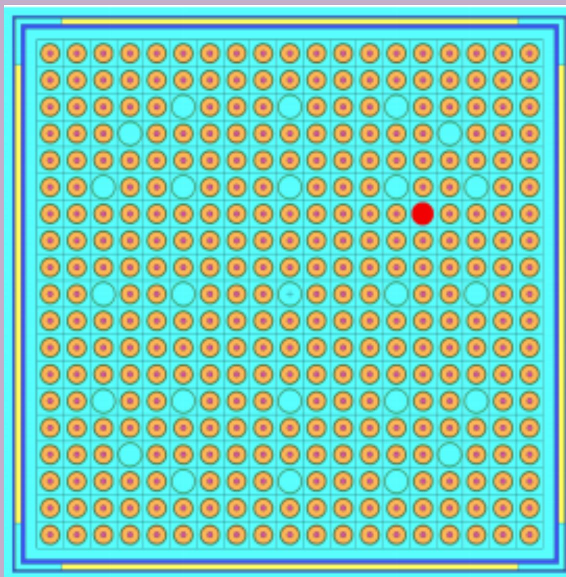
- The within-cell source distribution when calculating coefficients is an important source of error
- Use bounding values for radial and energy distributions
- Radial distribution
  - Uniform
  - Edge-only



- Energy distribution
  - U-235 spectrum
  - Pu-239 spectrum
  - Weighted spectrum



Fission Matrix Coefficients

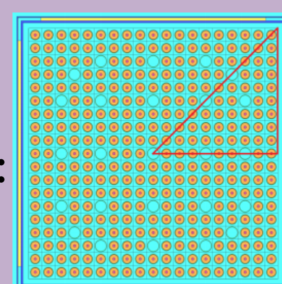


Source in single pin

0.03	0.04	0.03	0.05	0.06	0.10	0.11	0.14	0.18	0.19	0.23	0.24	0.27	0.30	0.29	0.27	0.26	0.29	0.31
0.03	0.03	0.05	0.08	0.10	0.12	0.16	0.16	0.20	0.28	0.28	0.30	0.39	0.42	0.45	0.39	0.33	0.30	0.27
0.03	0.04	0.06	0.09	0.11	0.00	0.20	0.22	0.28	0.00	0.35	0.43	0.51	0.00	0.54	0.57	0.44	0.38	0.30
0.05	0.06	0.09	0.00	0.14	0.17	0.19	0.25	0.27	0.38	0.45	0.46	0.55	0.69	0.73	0.00	0.65	0.43	0.35
0.04	0.06	0.09	0.12	0.15	0.18	0.23	0.26	0.33	0.45	0.50	0.56	0.66	0.77	0.76	0.82	0.73	0.51	0.41
0.04	0.08	0.00	0.11	0.16	0.00	0.25	0.29	0.39	0.00	0.52	0.60	0.75	0.00	1.02	0.92	0.00	0.65	0.46
0.04	0.06	0.09	0.09	0.13	0.20	0.24	0.28	0.37	0.49	0.51	0.61	0.71	0.98	**2.07**	0.91	0.73	0.60	0.43
0.03	0.06	0.09	0.08	0.13	0.17	0.22	0.28	0.32	0.40	0.49	0.59	0.71	0.80	0.85	0.82	0.67	0.53	0.39
0.05	0.05	0.08	0.10	0.14	0.18	0.20	0.28	0.31	0.43	0.51	0.54	0.62	0.72	0.72	0.70	0.64	0.51	0.40
0.05	0.05	0.00	0.09	0.13	0.00	0.21	0.22	0.33	0.00	0.44	0.48	0.60	0.00	0.61	0.59	0.00	0.48	0.31
0.04	0.04	0.07	0.09	0.12	0.15	0.19	0.20	0.26	0.35	0.38	0.38	0.44	0.53	0.48	0.52	0.47	0.44	0.26
0.03	0.05	0.06	0.09	0.09	0.14	0.17	0.19	0.22	0.25	0.30	0.33	0.38	0.46	0.40	0.41	0.40	0.35	0.23
0.03	0.04	0.06	0.08	0.09	0.11	0.14	0.15	0.18	0.24	0.26	0.27	0.30	0.39	0.36	0.33	0.32	0.30	0.20
0.03	0.04	0.00	0.07	0.09	0.00	0.11	0.14	0.18	0.00	0.27	0.23	0.28	0.00	0.31	0.29	0.00	0.25	0.16
0.03	0.04	0.05	0.07	0.06	0.08	0.10	0.12	0.17	0.18	0.16	0.19	0.25	0.26	0.24	0.23	0.23	0.19	0.12
0.02	0.03	0.04	0.00	0.06	0.06	0.08	0.09	0.15	0.14	0.15	0.15	0.16	0.22	0.20	0.00	0.16	0.13	0.09
0.02	0.02	0.03	0.04	0.07	0.00	0.06	0.07	0.11	0.00	0.13	0.12	0.14	0.00	0.15	0.16	0.13	0.10	0.08
0.01	0.02	0.02	0.03	0.03	0.05	0.05	0.06	0.07	0.10	0.09	0.10	0.10	0.11	0.10	0.09	0.10	0.09	0.06
0.01	0.01	0.02	0.03	0.02	0.03	0.05	0.04	0.06	0.05	0.06	0.09	0.06	0.06	0.07	0.06	0.06	0.07	0.07

Neutron production tallied in all cells (results are * 100)

Repeat for all 49 source pins in octant:



Total precalculation time (2 materials, 49 source pins each): 4600 min

Geometric similarity of coefficients

- Previous calculation only gives 49 rows of the fission matrix
- Obtain the rest using octal symmetry and geometric similarity
- Axially, translate the coefficients calculated from the center

Pre-calculation of Coefficients

- For a range of burnup and cooling times, the 49 source calculations are repeated
  - ~50 minutes per calculation, ~2500 minutes per material (in serial)
- These values are interpolated for any possible burnup and cooling time in the pool
- If the pool is shuffled (or misplaced), precalculations do NOT need to be repeated

Solution of Fission Matrix Equations

- Once the 49 base sets of fission matrix coefficients have been calculated, the full set of coefficients is obtained using the geometric considerations for any pool configuration.
- $N = 336 * 144 * N_{assemblies}$ total equations
- Solve for k using a power iteration

$$\vec{F}^{(m+1)} = \frac{1}{k} A\vec{F}^{(m)}$$

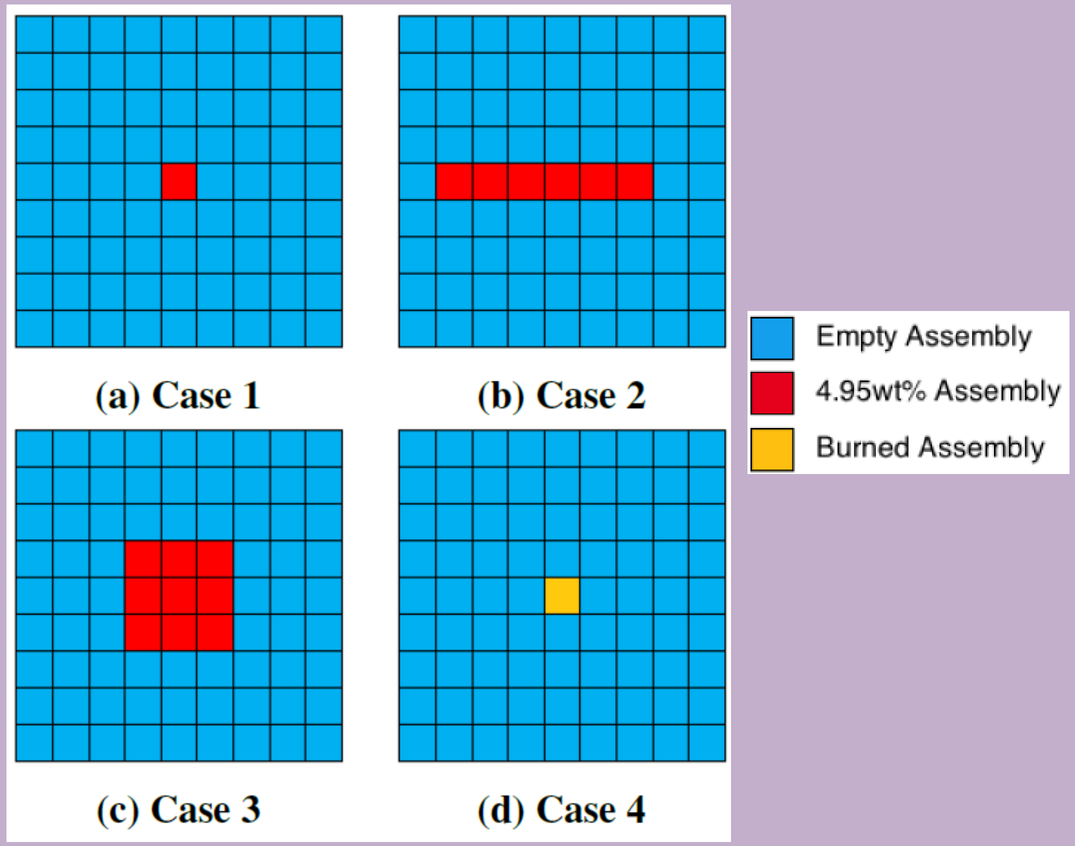
- Subcritical multiplication can be solved given the independent source distribution

$$\vec{F}^{(m+1)} = A\vec{F}^{(m)} + B\vec{S}$$

- When two different materials are present, use the coefficients that were calculated for the destination cell

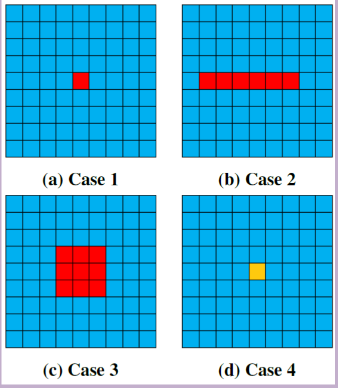
Test Cases

- Four test problems
- Fresh 4.95% enriched fuel
- Burned assembly at 15 GWd/MTU
- Reference results with MCNP6
  - Fixed source: 1e7 histories
  - Eigenvalue: 40k histories/cycle, 400 skipped cycles, 500 active cycles



Fresh Fuel Eigenvalue Results

- Good agreement between MCNP and FM method
- Much faster results with FM
- ~150 pcm difference between spatial source distributions
- Computation time goes up for larger problems in FM (proportional to N), but not for MCNP (since using the same number of histories)

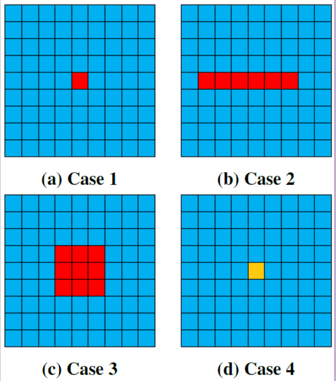


(a) Case 1 (b) Case 2 (c) Case 3 (d) Case 4

Case	MCNP			FM Uniform Source		FM Radial Source		Speedup
	k	1σ [pcm]	Time [min]	k	Rel. Diff. [pcm]	k	Rel. Diff. [pcm]	
1	0.79015	30	4242	0.79032	21	0.78916	-125	1547
2	0.83082	18	5200	0.83061	-26	0.82948	-161	331
3	0.86095	18	4116	0.86023	-84	0.85903	-224	167

Fresh Fuel Fixed-Source

- Radial source is much worse
  - Poor representation of the total source – the independent source was assumed to be uniform in the cell
- Excellent speedup values
- Much more information is obtained (3-d pinwise fission rate)



(a) Case 1 (b) Case 2 (c) Case 3 (d) Case 4

Case	MCNP			FM Uniform Source		FM Radial Source		Speedup
	M	1σ [pcm]	Time [min]	M	Rel. Diff. [pcm]	M	Rel. Diff. [pcm]	
1	3.33244	70	15987	3.34026	235	3.31778	-440	5769
2	4.30842	70	21356	4.32342	348	4.28620	-516	666
3	5.42369	80	27783	5.41735	-117	5.36189	-1139	537

Spent Fuel Eigenvalue Results

- Single assembly at 15 MWd/MTU
- Added energy spectrum in addition to radial source distribution
- 2000x speedup (~2.5 min) for a few hundred pcm
- The highest and lowest range of assumptions almost bracket the MCNP value

MCNP Reference

k	1σ - [pcm]	Time - [min]
0.69794	18	5577

Fission Matrix

Spatial Source Distribution	Spectrum	k	Rel. Diff., MCNP [pcm]	Speedup
Uniform	^{235}U	0.70287	706	2067
	^{239}Pu	0.69951	225	2063
	Weighted	0.70244	645	2058
Radial	Weighted	0.70059	379	2041
	^{239}Pu	0.69855	87	2123

Spent Fuel Fixed-Source Results

- Added energy spectrum in addition to radial source distribution
- 4000x speedup (~2.5 min)
- M values are worse than k, but this is less critical

MCNP Reference

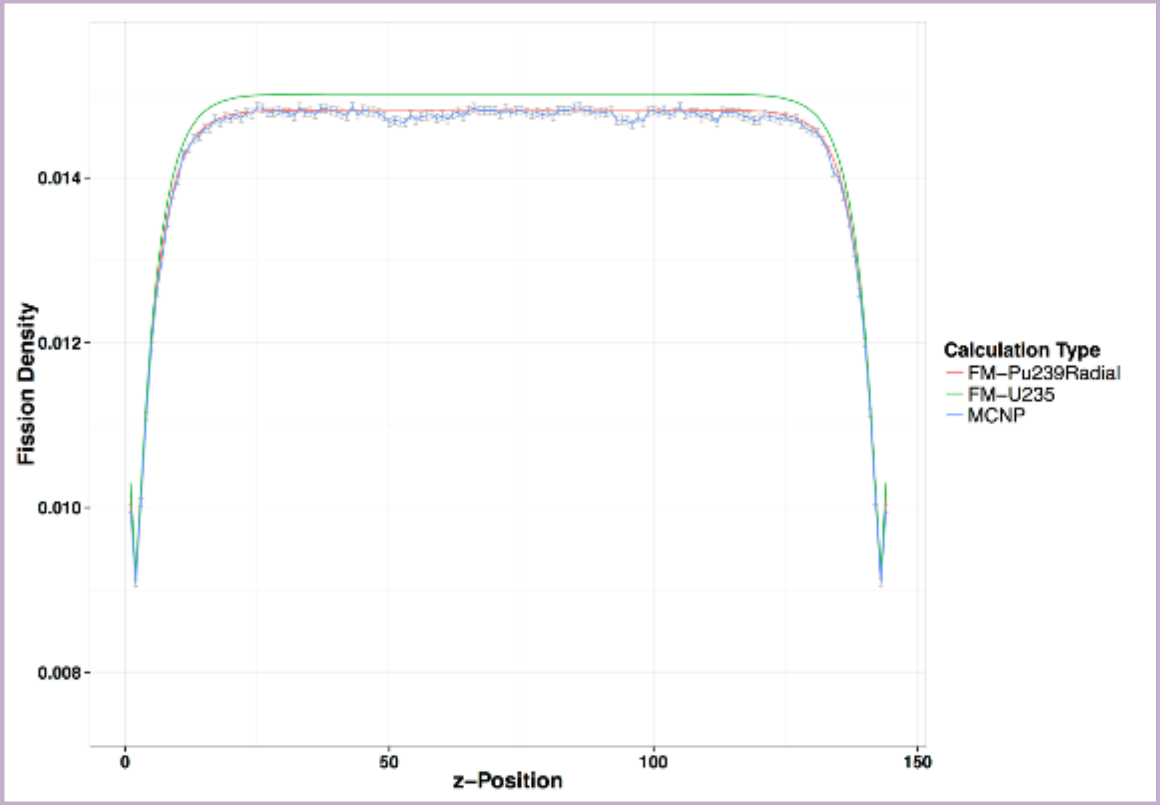
M	1σ - [pcm]	Time - [min]
2.06625	60	10528

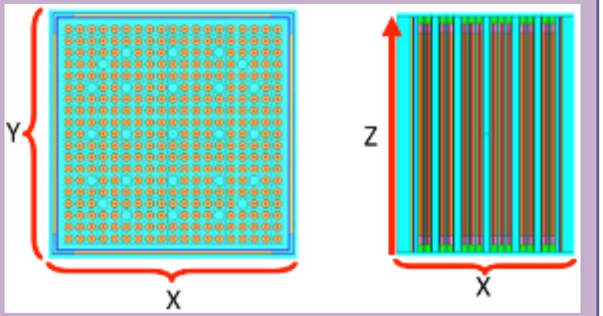
Fission Matrix

Spatial Source Distribution	Spectrum	M	Rel. Diff., MCNP [pcm]	Speedup
Uniform	^{235}U	2.09951	1610	3897
	^{239}Pu	2.07787	562	3871
	Weighted	2.09681	1479	3906
Radial	Weighted	2.08469	893	3879
	^{239}Pu	2.07250	302	4002

Fission density plots

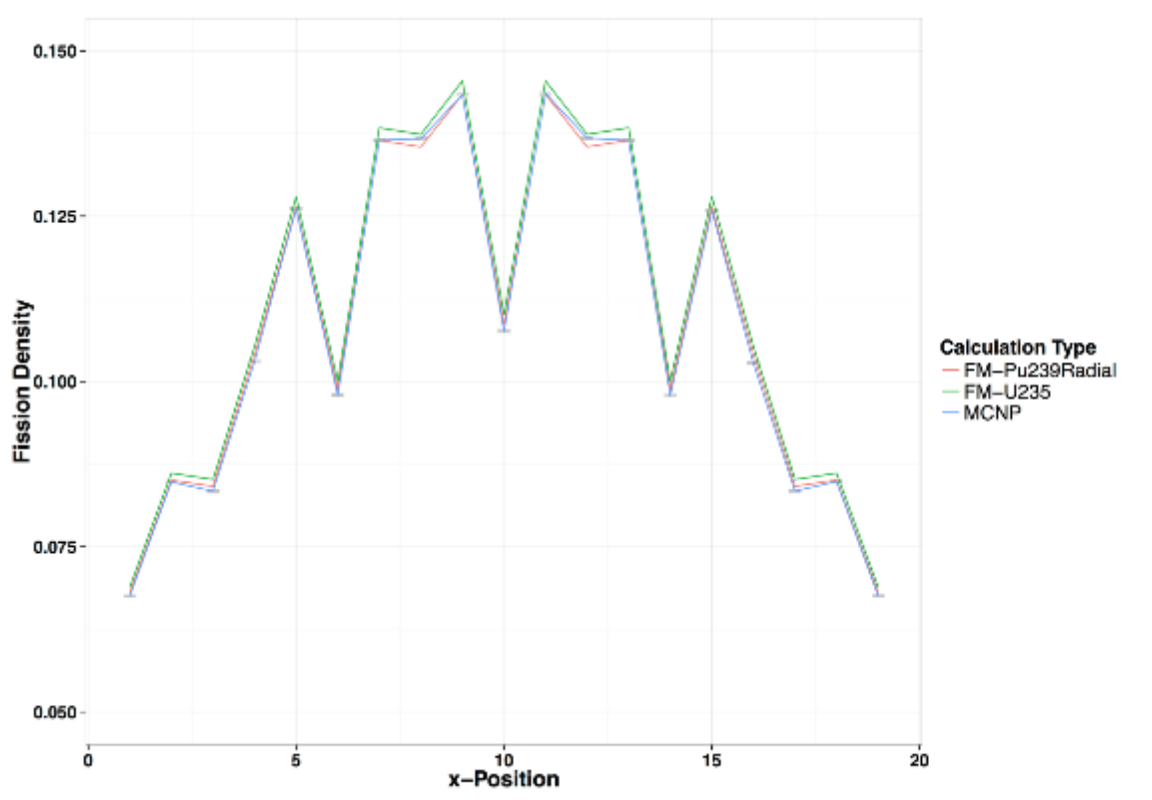
- Axial fission density for entire assembly (x-y integrated)

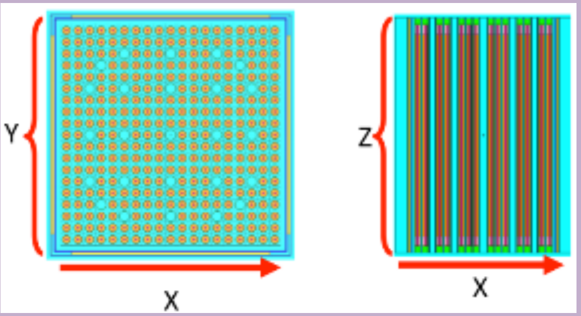




Fission density plots

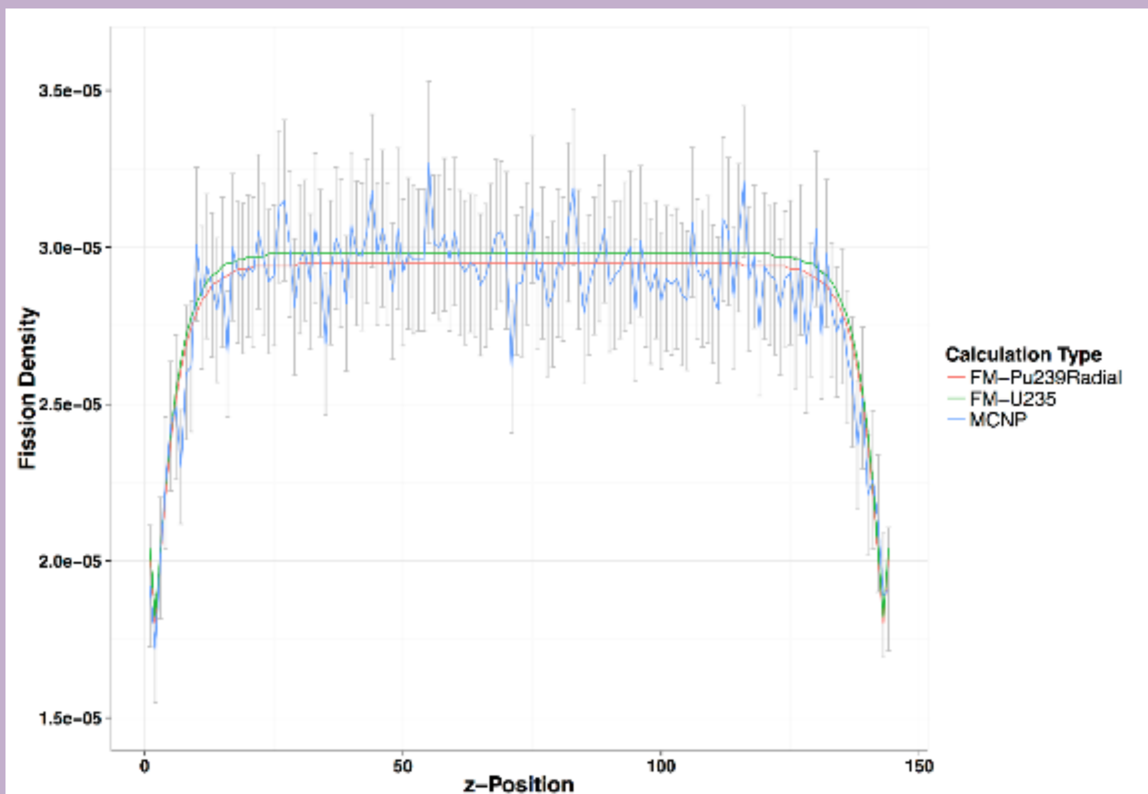
- *x*-fission density for entire assembly (*y*-*z* integrated)

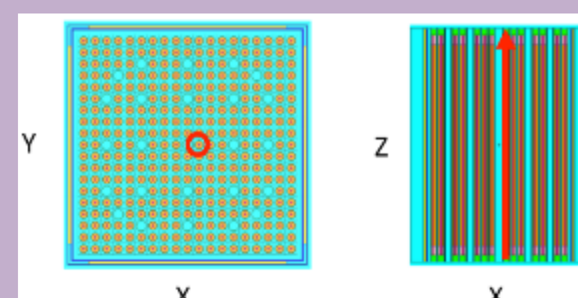




Fission density plots

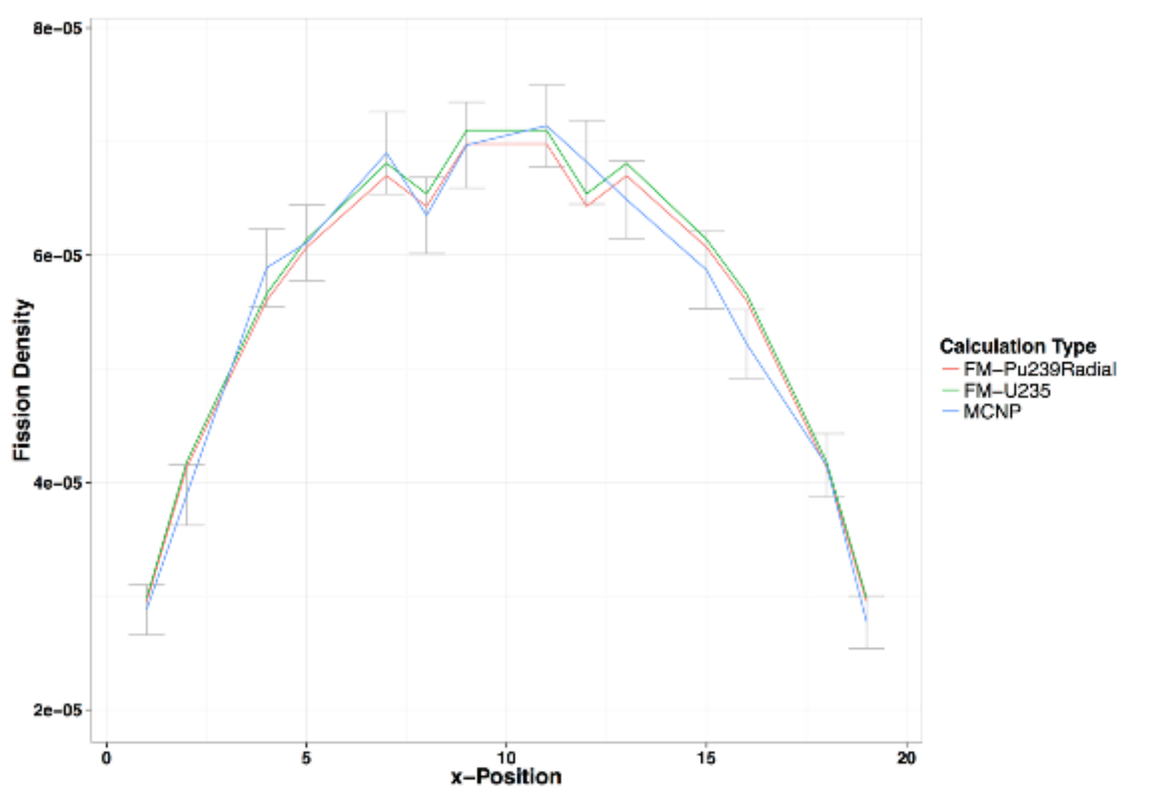
- Axial fission density for a single pin (x=10, y=11)

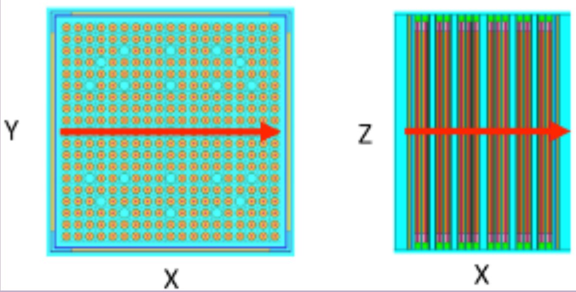




Fission density plots

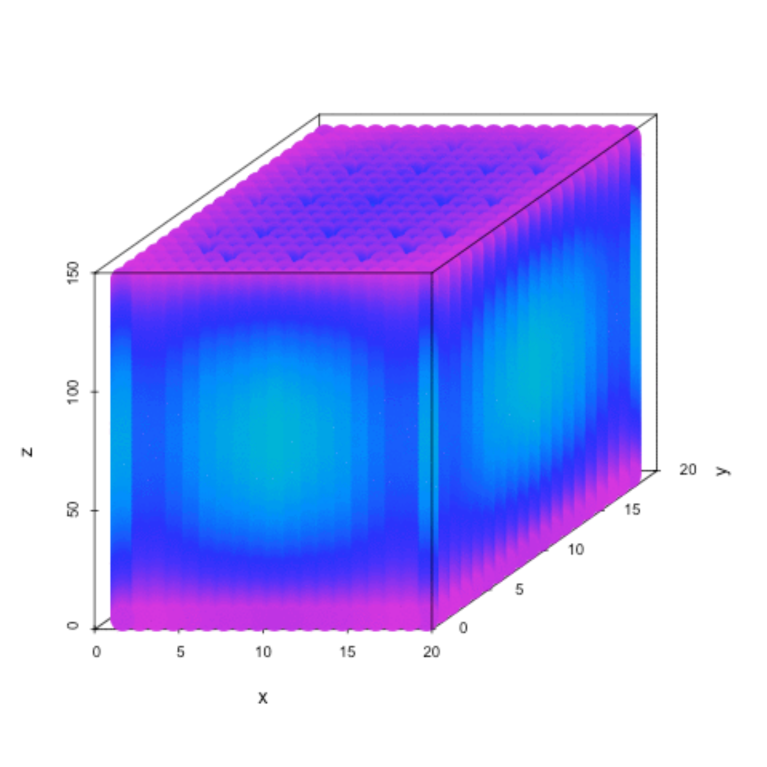
- x-fission density for a single z-level (y=10, z=72)



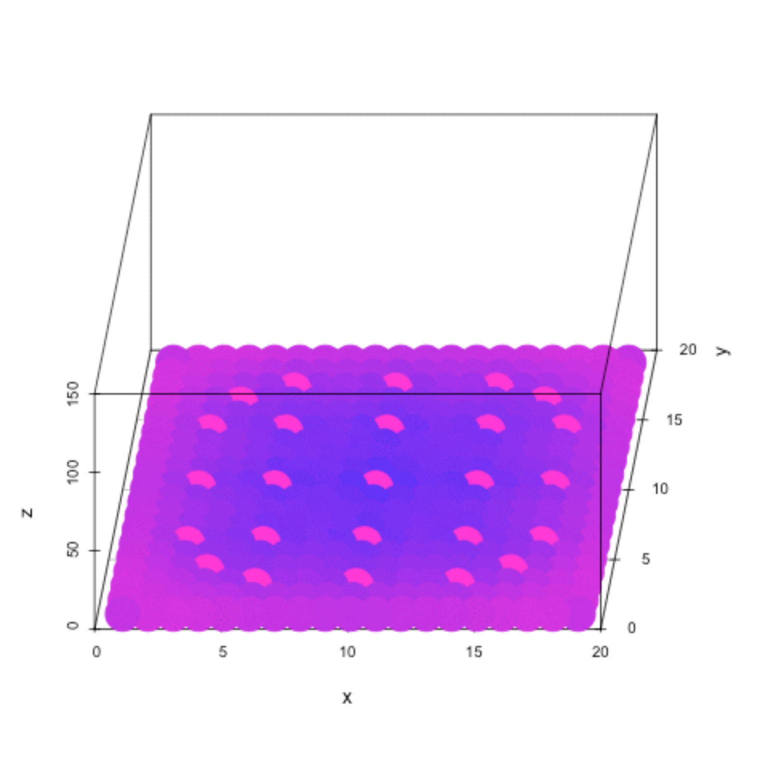


3-D Fission Density

Y-level Animation



Z-level Animation



Conclusions

- Our pre-calculated fission matrix method provides a very fast tool to obtain accurate neutronics calculations in a spent fuel pool
- Once pre-calculations are done, additional calculations can be performed very quickly
- Pin-wise, axial fission rates can be obtained (Monte Carlo is very slow for this)
- Work is being to obtain more accurate and faster results