

MRT Methodologies for Application to Nuclear Safeguards, Safety and Security

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Virginia Tech Transport Theory Group (VT³G)

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Completed June 2011

Energy Seminar Series

Nuclear Science and Engineering Program, The Colorado School of Mines, Golden Co, April 6-7, 2016

Nuclear Science and Engineering Lab (NSEL) @ Arlington

NSEL at Arlington Operates under auspices of ICTAS* and Mechanical Engineering Department. It engages with various entities/organizations at Virginia Tech and beyond to address different applications including power, security, medicine, and policy (<http://nsel.ncr.vt.edu>)




*Institute of Critical Technology and Applied Science

Collaborations @ Virginia Tech

Virginia Tech	Activity	Campus
Physics Department	Neutrino Physics Center, GEM*STAR initiative	Blacksburg
Nuclear Engineering Program	Education & research	Blacksburg
Discovery Analytics Center	Inference and detection	NCR
Hume Center for national security	Cyber security	NCR
School of Public and International Affairs (SPIA) & Department of Science and Technology in Society	Nuclear nonproliferation and policy	NCR

Collaborations with other organizations

Organization	Activity	Location
US Naval Academy (USNA)	<ul style="list-style-type: none"> ➤ Signed a research and education partnership, Aug 2015 ➤ Initiated benchmarking of the RAPID code system using USNA's subcritical facility (for nuclear safeguards) ➤ Discussing establishing a special graduate program for USNA graduates 	Annapolis, MD
Naval Surface Warfare Center, Carderock	Tandem linear accelerator research; small modular reactor use in military	MD
Federation of American Scientist	Workforce on LEU nuclear fueled naval vessels	DC
Georgia Tech (lead) with 10 other organizations including VT	Design of Integral Inherently Safe LWR reactor system design	
George Washington University	Nuclear education; GEM*STAR	
Oak Ridge National Lab	GEM*STAR, spent fuel casks	Tennessee
Collaboration among NE, Physics & MSE	Safe, Secure, Sustainable Nuclear Power (S3NPower)	Blacksburg, ICTAS



<http://www.virginianuclear.org/>

Formation of VNEC nonprofit organization

Organization	Activities	Location
Virginia Nuclear Energy Consortium (VNEC)	<ul style="list-style-type: none">• Promotion of nuclear industry, education and research• Membership include: AREVA, B&W, Dominion, GE, Newport News Shipbuilding, UVA, VCU, and VT• Prof. Haghighat is Chairman of the Board	Virginia

NSEL – Organization of Workshops/Forums

Year (date)	Title
2011 (Nov 7-11)	13 th International Workshop on Particle Transport Simulation of Nuclear Systems (http://www.cpe.vt.edu/transport)
2012 (March 11-12)	Symposium on Low Power Critical Facilities (LPCF) in collaboration with SUNRISE* (http://www.cpe.vt.edu/lpcf)
2012 (Nov 5)	Forum on Nuclear Regimes: Future Outlooks; sponsors included AREVA, ICTAS, VT-NCR, and partners included Naval Postgraduate school, Federation of American Scientists, and George Washington's Elliot College of International Affairs (http://www.ictas.vt.edu/nuclear)
2013 (Aug 7)	Seminar on nuclear power & education for a group of international reporters (at the request of Department of State) (http://nsel.ncr.vt.edu)
2014 (July 20)	a half-day workshop on "Advanced particle transport methodologies/tools for nuclear safeguards and non-proliferation," INMM 55 th Annual Meeting, Atlanta, Georgia. (In collaboration with Georgia Tech)
2014 (Sept 28)	A half-day workshop on "Hybrid particle transport methods for solving complex problems in real-time," PHYSOR 2014 International Conference, Kyoto, Japan. (In collaboration with Georgia Tech)
2014 (Dec 15-18)	MRT Methodologies for Real-Time Simulation of Nuclear Safeguards & Nonproliferation Problems,' Modeling and Simulation for Safeguards and Nonproliferation <i>Workshop ORNL.</i>
2015 (June 23-25)	1 st Workshop on Methodologies for Spent Nuclear Fuel Pool Simulations (Safety and Safeguards) (http://www.cpe.vt.edu/nuclear)

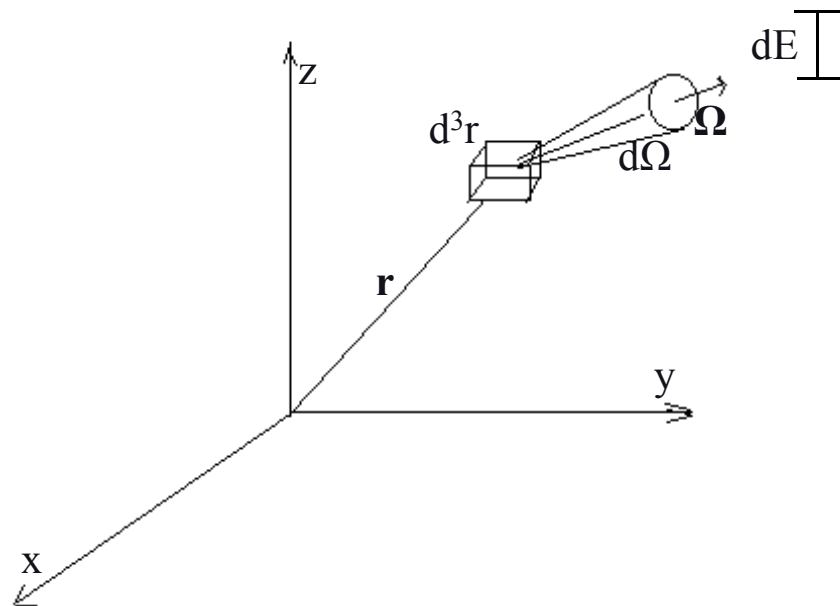
*Southeast Universities Nuclear Reactors Institute for Science and Education

Particle Transport Theory

Objective

Determine the expected number of particles in a phase space $(d^3rdEd\Omega)$ at time t :

$$n(\vec{r}, E, \hat{\Omega}, t)d^3rdEd\Omega$$



Number density is used to determine angular flux/current, scalar flux and current density, partial currents, and reaction rates.

Simulation Approaches

- **Deterministic Methods**

- Solve the linear Boltzmann equation to obtain the expected flux in a phase space

- **Statistical Monte Carlo Methods**

- Perform particle transport experiments using random numbers (RN's) on a computer to estimate average properties of a particle in phase space

Deterministic – Linear Boltzmann Equation

- **Integro-differential form**

$$\begin{aligned}
 & \text{streaming} \quad \hat{\Omega} \cdot \nabla \Psi(\vec{r}, E, \hat{\Omega}) + \sigma(\vec{r}, E) \Psi(\vec{r}, E, \hat{\Omega}) = \\
 & \int_0^{\infty} dE' \int_{4\pi} d\Omega' \sigma_s(\vec{r}, E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega}) \Psi(\vec{r}, E', \hat{\Omega}') + \\
 & \frac{\chi(E)}{4\pi} \int_0^{\infty} dE' \int_{4\pi} d\Omega' \nu \sigma_f(\vec{r}, E') \Psi(\vec{r}, E', \hat{\Omega}') + S(\vec{r}, E, \hat{\Omega}) \\
 & \text{collision} \quad \text{scattering} \quad \text{fission} \quad \text{Independent source}
 \end{aligned}$$

- **Integral form**

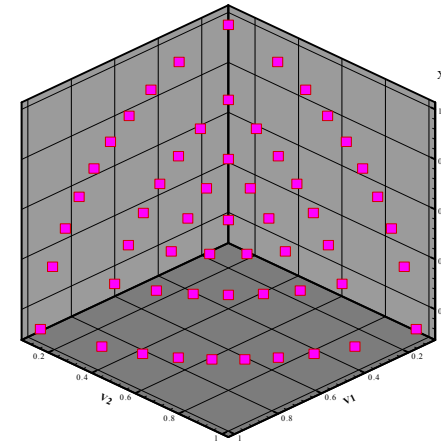
$$\psi(\vec{r}, E, \hat{\Omega}) = \int_0^R d|\vec{r} - \vec{r}'| Q(r') e^{-\tau_E(\vec{r}, \vec{r}')} + \psi(\vec{r}_s, E, \hat{\Omega}) e^{-\tau_E(\vec{r}, \vec{r}_s)}$$

Integro-differential - Solution Method

- **Angular variable: Discrete Ordinates (Sn) method:**

A discrete set of directions $\{ \hat{\Omega}_m \}$
and associated weights $\{ w_m \}$ are selected

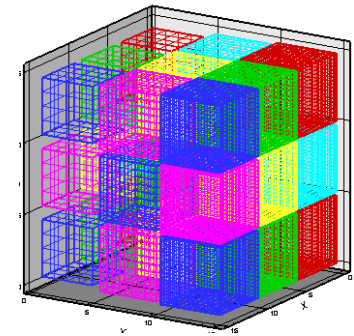
$$\hat{\Omega}_m \cdot \nabla \Psi(\vec{r}, E, \hat{\Omega}_m) + \sigma(\vec{r}, E) \Psi(\vec{r}, E, \hat{\Omega}_m) = q(\vec{r}, E, \hat{\Omega}_m)$$



- **Spatial variable**

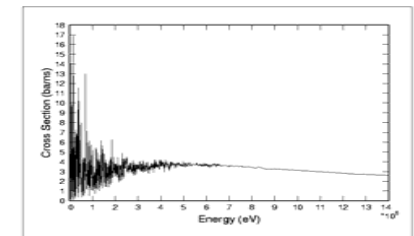
Integrated over fine meshes using FD or FE methods

$$\Psi_{m,g,A} = \frac{\int d^3r \Psi_{m,g}(\vec{r})}{\Delta V_{ijk}}$$



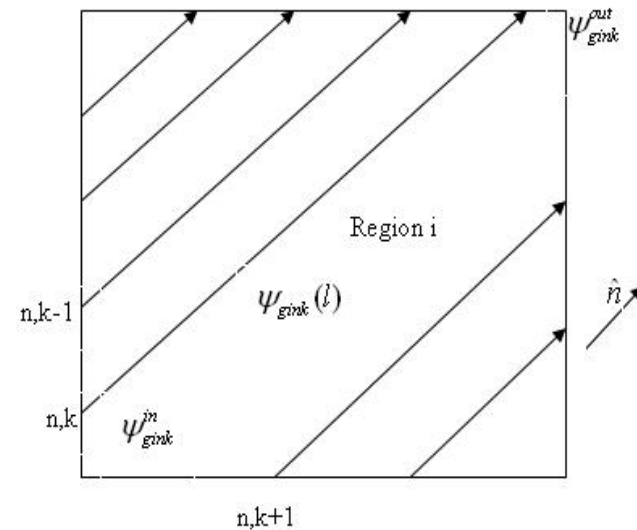
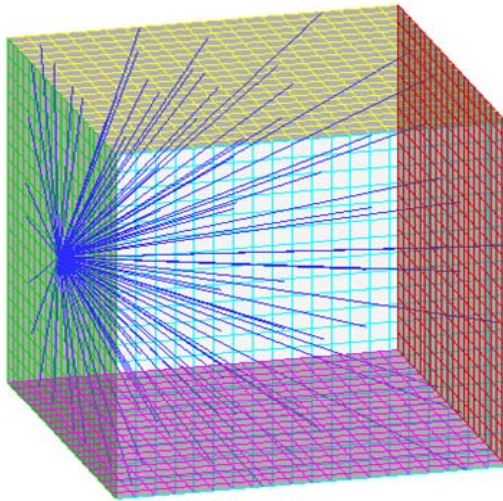
- **Energy variable**

Integrate over energy intervals to prepare multigroup cross sections, σ_g



Integral - Solution method

- **Method of Characteristic (MOC):** Model is partitioned into **coarse meshes** and transport equation is solved along the characteristic paths (k) (parallel to each discrete ordinate (n)), filling the mesh, and averaged



$$\psi_{g,m,i,k}(t_{m,i,k}) = \psi_{g,m,i,k}(0) \exp(-\sigma_{g,i} t_{m,i,k}) + \frac{Q_{g,m,i}}{\sigma_{g,i}} (1 - \exp(-\sigma_{g,i} t_{m,i,k}))$$

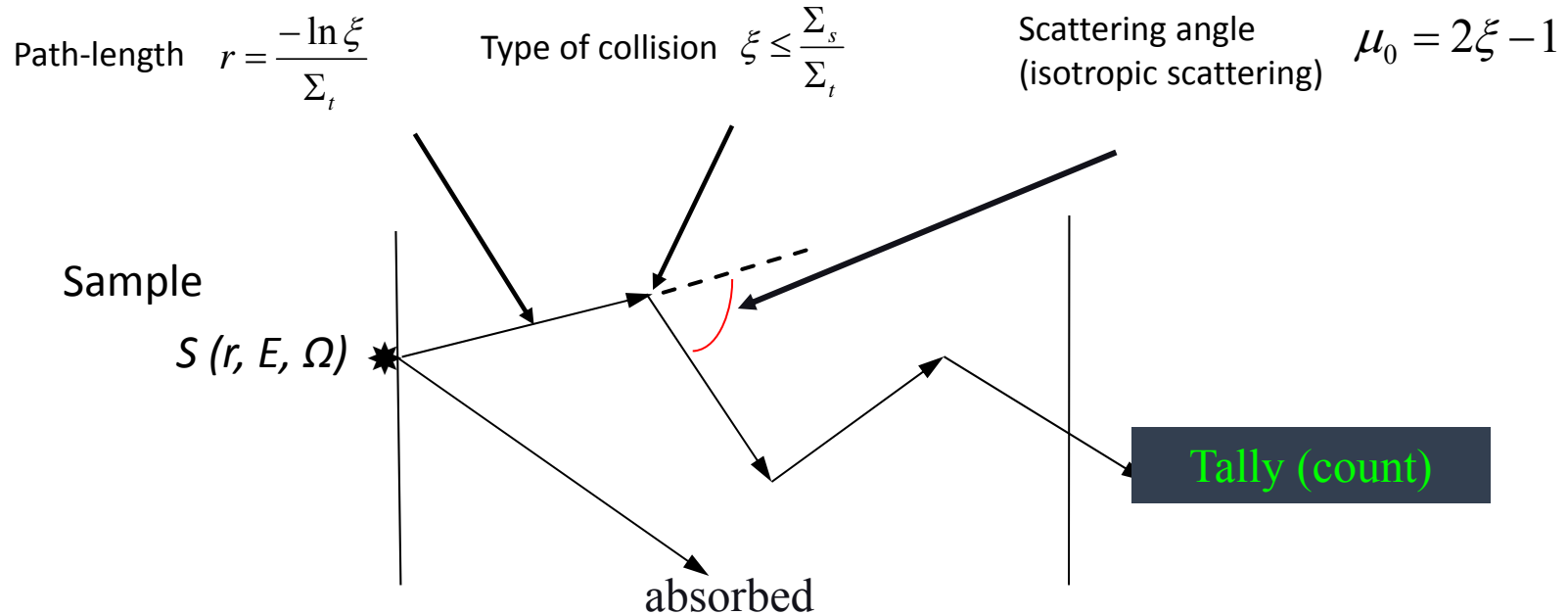
Deterministic - Issues/Challenges/Needs

- Robust numerical formulations (e.g., adaptive differencing strategy)
- Algorithms for improving efficiency (i.e., acceleration techniques – synthetic formulations and pre-conditioners)
- Use of advanced computing hardware & software environments
- Pre- and post-processing tools
- Multigroup cross section preparation
- Benchmarking

Over the past 29 years, VT³G address all the above issues

Monte Carlo Methods

- Perform an experiment on a computer; “exact” simulation of a physical process



Issues

- *Precise expected values; i.e., small relative uncertainty, $R_{\bar{x}} = \frac{\sigma_{\bar{x}}}{\bar{x}}$, requiring large computation time*
- *Therefore, Variance Reduction techniques are needed for real-world problems!*
- *For eigenvalue problems, the source convergence is an added difficulty.*

Deterministic vs. Monte Carlo

Item	Deterministic	MC
Geometry	Discrete/ Exact	Exact
Energy treatment – cross section	Discrete	Exact
Direction	Discrete/ Truncated series	Exact
Input preparation	Difficult	simple
Computer memory	Large	Small
Computer time	Small	Large
Numerical issues	Convergence	Statistical uncertainty
Amount of information	Large	Limited
Parallel computing	Complex	Trivial

Why not MC only?

- Because of the difficulty in obtaining **detail information** with **reliable statistical** uncertainty in a **reasonable time; examples are:**
 - Real-time simulations
 - Obtaining energy-dependent flux distributions,
 - Time-dependent simulations,
 - Sensitivity analysis,
 - Determination of uncertainties

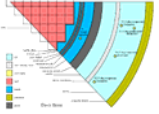
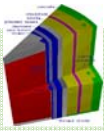
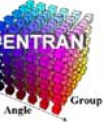
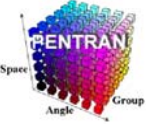
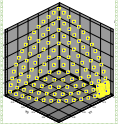
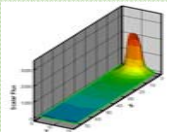
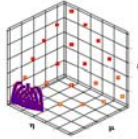
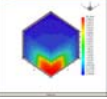

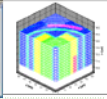



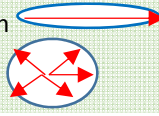
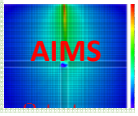
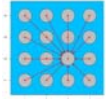

Why not use advanced hardware?

- VT³G has developed vector and parallel algorithms:
 - Developed two large codes: PENTRAN (1996) and TITAN (2004)

Why not use hybrid methods?

- **Deterministic-deterministic** (differencing schemes, different numerical formulations, generation of multigroup cross sections, generation of angular quadratures, acceleration techniques)
 - VT³G has developed various algorithms; a few have been implemented in PENTRAN and TITAN
- **Monte Carlo-deterministic** (variance reduction with the use of deterministic adjoint)
 - VT³G has developed CADIS, A³MCNP in 1997; *CADIS has become popular recently!*

VT³G Milestones & Contributing Current/Former Students (1986-2015)

1986-1989	<ul style="list-style-type: none"> Vector computing of 1-D Sn spherical geometry algorithm Development an adjoint methodology for simulation TMI-2 reactor 		Prof. Haghighat	
1989-1992	<ul style="list-style-type: none"> Vector and parallel processing of 2-D Sn algorithms Simulation of Reactor Pressure Vessel (RPV) 		Prof. R. Mattis, Pitt. Prof. B. Petrovic, GT	
1992-1994	<ul style="list-style-type: none"> Parallel processing of 2-D Sn algorithms & Acceleration methods Determination of uncertainties in the RPV transport calculations 		Dr. M. Hunter, W Prof. B. Petrovic, GT	
1994-1995	<ul style="list-style-type: none"> 3-D parallel Sn Cartesian algorithms Monte Carlo for Reactor Pressure Vessel (RPV) benchmark using Weight-window generator; deterministic benchmarking of power reactors 		Dr. G. Sjoden, DOD Dr. J. Wagner, ORNL	
1995-1997	<ul style="list-style-type: none"> Directional Theta Weight (DTW) differencing formulation PENTRAN (Parallel Environment Neutral Particle TRANsport) code system CADIS (Consistent Adjoint Driven Importance Sampling) formulation for Monte Carlo Variance Reduction A³MCNP (Automated Adjoint Accelerate MCNP) 		Dr. B. Petrovic Dr. G. Sjoden, DOD Dr. J. Wagner, ORNL	
1997-2001	<ul style="list-style-type: none"> Parallel Angular & Spatial Multigrid acceleration methods for Sn transport Hybrid algorithm for PGNNA device PENMSH & PENINP for mesh and input generation of PENTRAN Ordinate Splitting (OS) technique for modeling a x-ray CT machine 		Dr. V. Kucukboyaci, W Dr. B. Petrovi, GT Prof. Haghighat Prof. Hgahighat	
2001-2004	<ul style="list-style-type: none"> Simplified Sn Even Parity (SSn-EP) algorithm for acceleration of the Sn method RAR (Regional Angular Refinement) formulation Pn-Tn angular quadrature set FAST (Flux Acceleration Simplified Transport) PENXMSH, An AutoCad driven PENMSH with automated meshing and parallel decomposition CPXSD (Contributon Point-wise cross-section Driven) for generation of multigroup libraries 		Dr. G. Longonil, PNNL Dr. A. Patchimpattapong, IAEA Dr. A. Alpan, W	  
2004-2007	<ul style="list-style-type: none"> TITAN hybrid parallel transport code system & a new version of PENMSH called PENMSHXP ADIES (Angular-dependent Adjoint Driven Electron-photon Importance Sampling) code system 		Dr. C. Yi, GT Dr. B. Dionne, ANL	
2007-2011	<ul style="list-style-type: none"> INSPECT-S (Inspection of Nuclear Spent fuel-Pool Calculation Tool ver. Spreadsheet), a MRT algorithm TITAN fictitious quadrature set and ray-tracing for SPECT (Single Photon Emission Computed Tomography) FMBMC-ICEU (Fission Matrix Based Monte Carlo with Initial source and Controlled Elements and Uncertainties) 		W. Walters, PhD Cand. Dr. C. Yi, GT Dr. M. Wenner, W	
2011-2013	<ul style="list-style-type: none"> New WCOS (Weighted Circular Ordinated Splitting) Technique for the TITAN SPECT Formulation Adaptive Collision Source (ACS) for Sn transport AIMS (Active Interrogation for Monitoring Special-nuclear-materials), a MRT algorithm 		K. Royston, PhD Cand. W. Walters, PhD Cand.	
2014-2015	<ul style="list-style-type: none"> TITAN-SDM - includes Subgroup Decomposition Method for multigroup transport calculation TITAN-IR - TITAN with iterative image Reconstruction for SPECT RAPID - Real-time Analysis for spent fuel Pool <i>in situ</i> detection 		N. Roskoff, PhD Stud. K. Royston, PhD Cand. W. Walters, PhD Cand.	

Remarks

- Particle transport-based methodologies are need for real-time simulation
- Even '*Fast*' particle transport codes, with parallel and hybrid algorithms, are **slow because of large number of unknowns**

Development of Transport Formulations for Real-Time Applications

- *Physics-Based* transport methodologies are needed:
- Developed *Multi-stage, Response-function Transport (MRT) methodology*
 - Based on problem physics **partition** a problem into **stages** (sub-problems),
 - For each stage employ response method and/or adjoint function methodology
 - Pre-calculate response-function or adjoint-function using an accurate and fast transport code
 - Solve a linear system of equations to couple all the stages

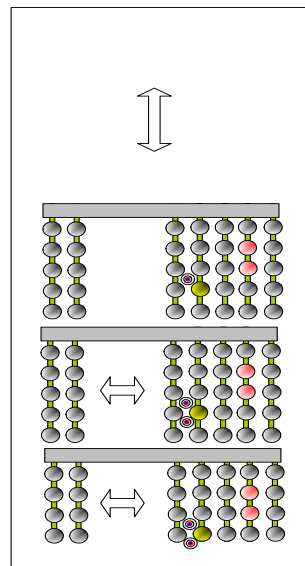
Examples for *MRT Algorithms*

- **Nondestructive testing:** Optimization of the Westinghouse's PGNNA active interrogation system for detection of RCRA (Resource Conservation and Recovery Act) (e.g., lead, mercury, cadmium) in waste drums (partial implementation of MRT; 1999)
- **Nuclear Safeguards:** Monitoring of spent fuel pools for detection of fuel diversion (2007) (funded by LLNL)
- **Nuclear nonproliferation:** Active interrogation of cargo containers for simulation of special nuclear materials (SNMs) (2013) (in collaboration with GaTech)
- **Spent fuel safety and security:** Real-time simulation of spent fuel pools for determination of eigenvalue, subcritical multiplication, and material identification (partly funded by I²S project, led by GaTech) (Ongoing)
- **Image reconstruction for SPECT (Single Photon Emission Computed Tomography):** Real-time simulation of an SPECT device for generation of project images using an MRT methodology and Maximum Likelihood Estimation Maximization (MLEM) (filed for a patent, June 2015)

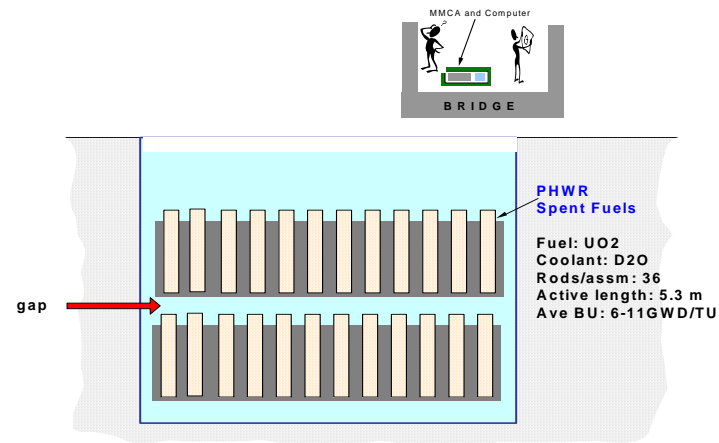
Nuclear Safeguards - Inspection of spent nuclear fuel pool

- **Goal:** Develop accurate and fast hybrid methodology and tool for inspection of spent fuel pool; funded by LLNL
- **Approach:** Use measurement and on-line computation to obtain trending curves

Atucha-1 Spent fuel pool



Top view of the pond



A front view of the pond that contains closely packed spent fuel assemblies

Issues

- Develop a fast and accurate computation tool which can estimate the detector response for various combinations of
 - Burnup
 - Cooling time
 - Pool lattice arrangement
 - Fuel type (enrichment)

MRT Methodology

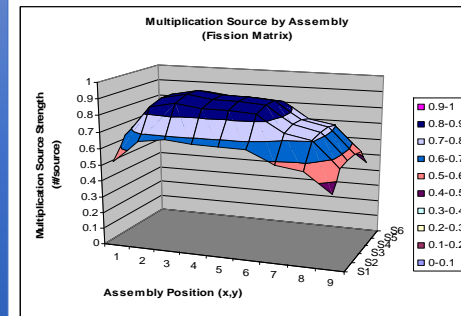
Online Calculation of detector response (R):

$$R_n = \langle S_n \phi_n^+ \rangle$$

Neutron source

Adjoint (Importance) function

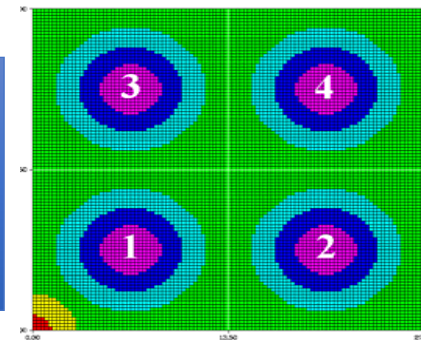
- **Source** ($S = S_{\text{intrinsic}} + S_{\text{subcritical-Multiplication}}$)
 - **Stage 1 - Intrinsic Source**
 - Spontaneous fission & (α, n) from fuel burnup calculation (ORIGEN-ARP)
- (Created a database)



- **Stage 2 - Subcritical Multiplication (Hybrid method)**
 - Simplified fission-matrix (FM) method
 - Use MCNP Monte Carlo to obtain $a_{i,j}$ for each pool type
- (Created a database for coef. a_{ij})

$$F_i = \sum_{j=1}^N a_{i,j} (F_j + S_j^{\text{int.}})$$

- **Adjoint function**
 - Stage 3 - Is obtained using the PENTRAN transport code
- (Created a database for multigroup adjoint for different lattice sizes)



Adjoint Function Methodology

- “Forward” Transport Equation

$$H\psi = q \quad \text{in } V$$

$$\psi = 0 \quad \text{on } \Gamma \text{ for } \hat{n} \cdot \hat{\Omega} < 0$$

where

$$H = \hat{\Omega} \cdot \nabla + \sigma_t(\vec{r}, E) - \int_0^\infty dE' \int_{4\pi} d\Omega' \sigma_s(\vec{r}, E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega})$$

- “Adjoint” Transport Equation

$$H^+\psi^+ = q^+ \quad \text{in } V$$

$$\psi^+ = 0 \quad \text{on } \Gamma \text{ for } \hat{n} \cdot \hat{\Omega} > 0$$

where

$$H^+ = -\hat{\Omega} \cdot \nabla + \sigma_t(\vec{r}, E) - \int_0^\infty dE' \int_{4\pi} d\Omega' \sigma_s(\vec{r}, E \rightarrow E', \hat{\Omega} \rightarrow \hat{\Omega}')$$

Adjoint function methodology – Detector response

Forward
approach

$$R = \langle \sigma_d \psi \rangle = \int_{V_d} dV \int_0^\infty dE \int_{4\pi} d\Omega \sigma_d(\vec{r}, E) \psi(\vec{r}, E, \hat{\Omega})$$

- The “commutation relation” between the “forward” and “adjoint” transport equations

$$\langle \psi^+ H \psi \rangle - \langle \psi H^+ \psi^+ \rangle = \langle \psi^+ q \rangle - \langle \psi q^+ \rangle$$

0 ←

- Then, $\langle \psi q^+ \rangle = \langle \psi^+ q \rangle$

- If we consider $q^+ = \sigma_d$

$$R = \langle \psi^+ q \rangle$$

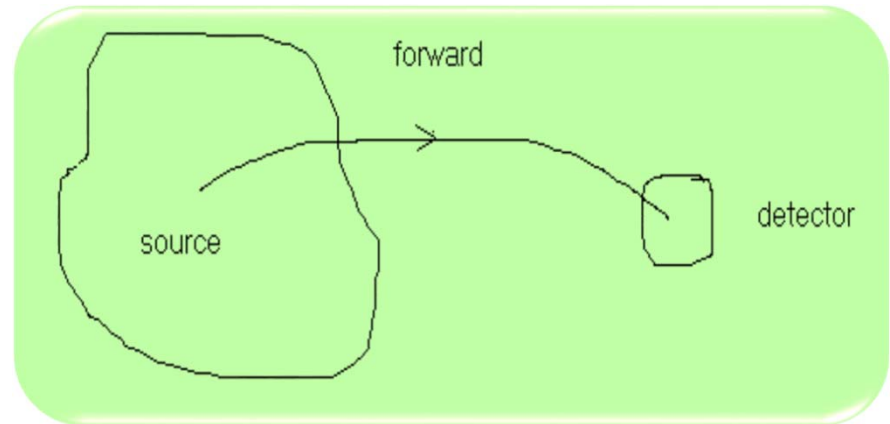
Demonstration

Standard

$$R = \langle \sigma_d \phi \rangle$$

Where,

$$H\phi = S$$

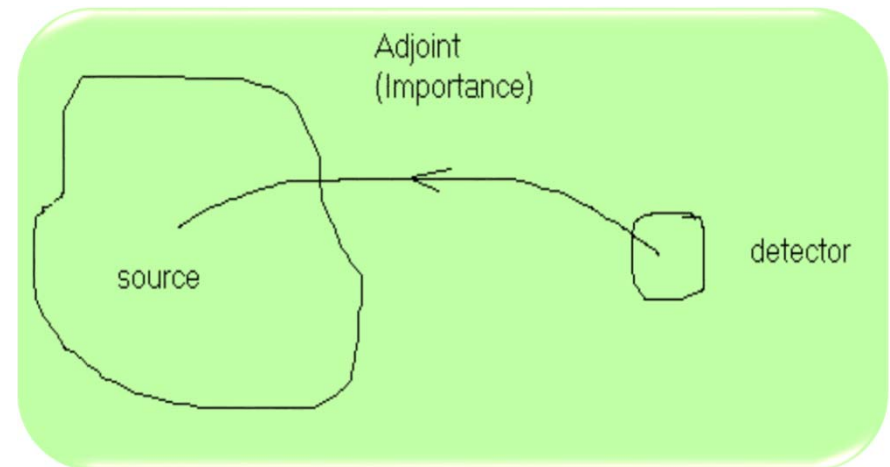


Adjoint Methodology

$$R = \langle S\phi^+ \rangle$$

Where,

$$H^+\phi^+ = \sigma_d$$



Derivation of Fission Matrix (FM) Formulation

- Eigenvalue formulation in operator form is expressed by

$$H\psi(\bar{p}) = \frac{1}{k} F\psi(\bar{p})$$

Where,

$$\bar{p} = (\bar{r}, E, \hat{\Omega})$$

$$H = \hat{\Omega} \cdot \nabla + \sigma_t(\bar{r}, E) - \int_0^\infty dE' \int_{4\pi} d\Omega' \sigma_s(\bar{r}, E' \rightarrow E, \mu_0)$$

$$F = \frac{\chi(E)}{4\pi} \int_0^\infty dE' \int_{4\pi} d\Omega' \nu \sigma_f(\bar{r}, E')$$

FM Derivation (cont)

- We may rewrite above equation as

$$S(\bar{p}) = \frac{1}{k} AS(\bar{p})$$

Where,

$$S = \tilde{F}\psi, \quad A = \tilde{F}H^{-1}\chi, \quad \&$$

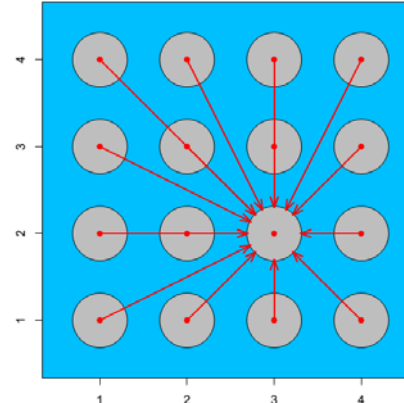
$$\tilde{F} = \frac{1}{4\pi} \int_0^\infty dE' \int_{4\pi} d\Omega' v\sigma_f(\bar{r}, E')$$

Fission Matrix (FM) Formulation

- Eigenvalue

$$F_i = \frac{1}{k} \sum_{j=1}^N a_{i,j} F_j$$

- k is eigenvalue
- F_j is fission source, S_j is fixed source in cell j
- $a_{i,j}$ is the number of fission neutrons produced in cell i due to a fission neutron born in cell j .



- Subcritical multiplication

$$F_i = \sum_{j=1}^N (a_{i,j} F_j + b_{i,j} S_j^{Intrinsic}),$$

$$M = \frac{\sum_{j=1}^N (F_j + S_j^{intrinsic})}{\sum_{j=1}^N S_j^{intrinsic}}$$

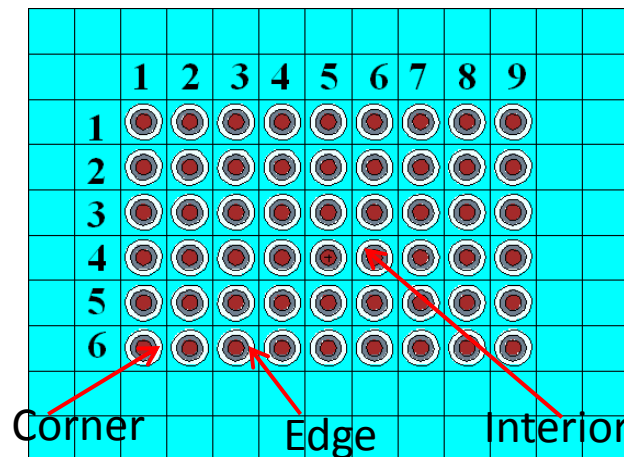
- $b_{i,j}$ is the number of fission neutrons produced in cell i due to a source neutron born in cell j .

Fission Matrix Coefficients – Inspection of Pool

- For this safeguards application, we have demonstrated that within the expected tolerance, the $b_{i,j}$ coefficients are equivalent to $a_{i,j}$, therefore, subcritical multiplication fission density is expressed by

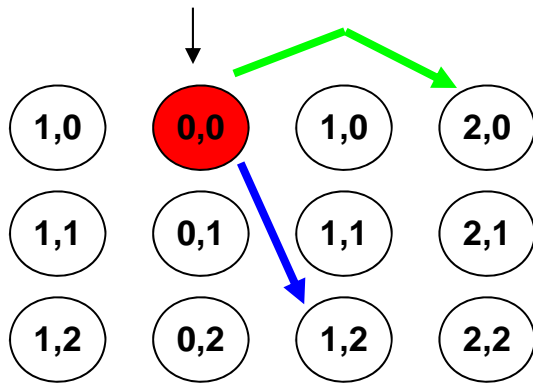
$$F_i = \sum_{j=1}^N a_{i,j} (F_j + S_j)$$

- Further, we have demonstrated that again within the tolerance, we need only three sets of coefficients depending on the position of assemblies, i.e., corner, edge, and interior



Calculation of FM coefficients

Source Assembly



Fission Matrix Coefficients			
	x-distance from source assembly		
y-distance	0	1	2
0	2.13E-01	4.98E-02	2.70E-03
1	4.56E-02	1.38E-02	1.22E-03
2	2.18E-03	1.11E-03	

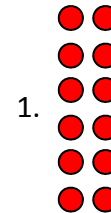
Coefficients for corner, edge and interior assemblies are within 1%

Hence, this finding reduces the necessary calculations to only one assembly location for different burnups and cooling time

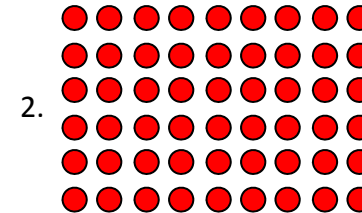
Testing the Simplified FM Methodology

Four test spent fuel scenarios

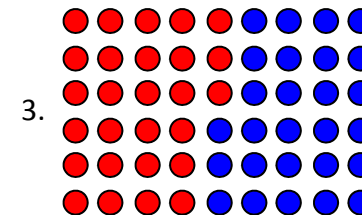
- 2x6 array, uniform source



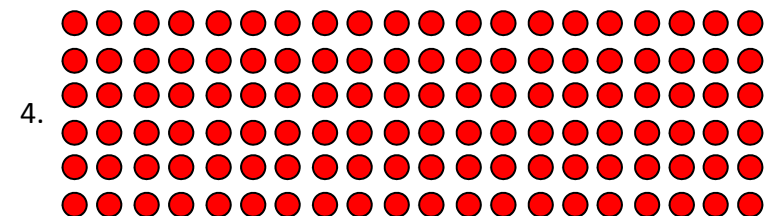
- 9x6 array, uniform source



- 9x6 array, 27 assemblies on the left with source strength 1, the rest with source strength 0.5



- 20x6 array, uniform source



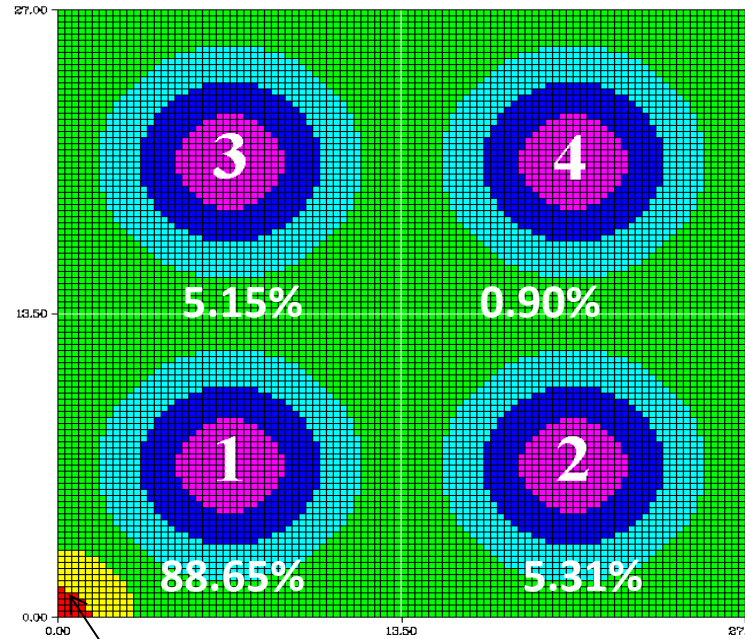
FM Testing Results

- Excellent agreement with Monte Carlo (<1%)
- Very fast
 - <1s for Fission-matrix method
 - ~1hr for Monte Carlo

Assembly Arrangement Case	M (MCNP)	M (Fission Matrix)	Difference	MCNP Uncertainty 1-σ
2x6, uniform	1.7133	1.7104	-0. 29%	0.0008
9x6, uniform	1.9988	1.9966	-0. 22%	0.0007
9x6, non-uniform	2.0033	1.9968	-0.65%	0.0013
20x6, uniform	2.0513	2.0444	-0. 69%	0.0012

Detector FOV

$$FR_i = \frac{\sum \psi_{ig}^* S_{ig} V_i}{\sum_j \sum_g \psi_{jg}^* S_{jg} V_j}$$



Fission Chamber (94 w% U-235)

INSPCT-S

(Inspection of Nuclear Spent fuel-Pool Computing Tool –Spreadsheet)

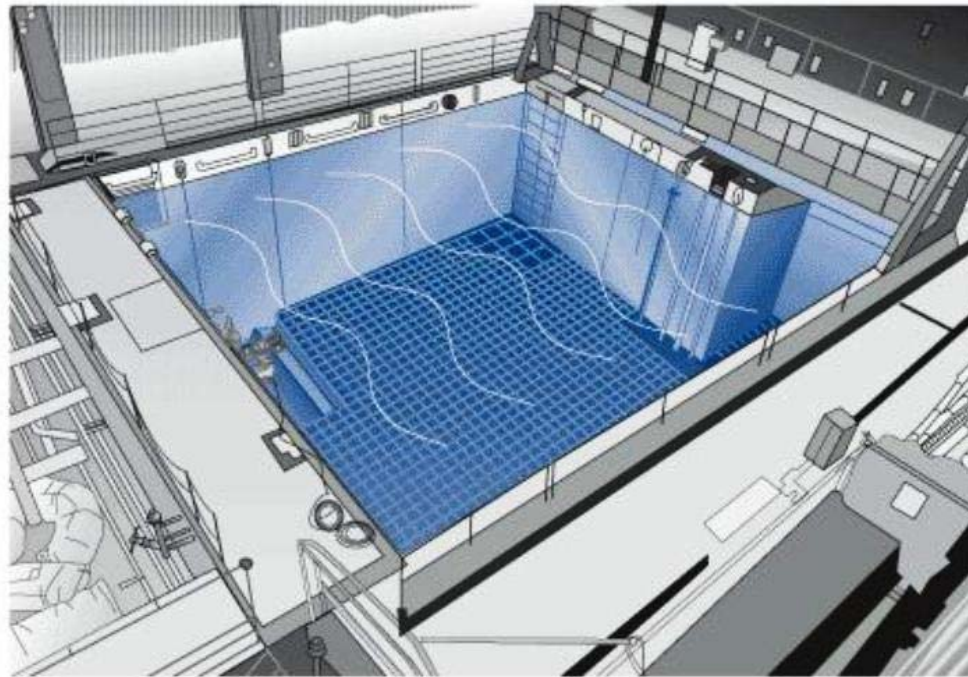
INSPCT-S solves

$$R_n = \langle S_n \phi_n^+ \rangle$$

INPUT										OUTPUT									
		src file	C:\Users\lali\Documents\haghD\ufttg\LLNL\INSPCT-s\se.dsrc																
COLUMNS		8 fm file	C:\Users\lali\Documents\haghD\ufttg\LLNL\INSPCT-s\se							Response Tolerance		5.28E-10							
ROWS		6 imp file	C:\Users\lali\Documents\haghD\ufttg\LLNL\INSPCT-s\se							15.00%									
Bumup										Independent Source									
(x,y)	1	2	3	4	5	6	7	8		(x,y)	1	2	3	4	5	6	7	8	
1	9000	9000	9000	9000	9000	9000	9000	9000		1	4.60E+07	3.39E+07	2.84E+07	2.48E+07	2.21E+07	1.94E+07	1.56E+07	13036948	
2	10000	10000	10000	10000	10000	10000	10000	10000		2	6.89E+07	5.30E+07	4.49E+07	3.86E+07	3.39E+07	2.91E+07	2.26E+07	18101692	
3	11000	11000	11000	11000	11000	11000	11000	11000		3	1.00E+08	8.04E+07	6.86E+07	5.84E+07	5.06E+07	4.29E+07	3.23E+07	25047256	
4	12000	12000	12000	12000	12000	12000	12000	12000		4	1.42E+08	1.17E+08	1.01E+08	8.51E+07	7.33E+07	6.15E+07	4.53E+07	34204842	
5	13000	13000	13000	13000	13000	13000	13000	13000		5	1.98E+08	1.67E+08	1.45E+08	1.22E+08	1.04E+08	8.67E+07	6.28E+07	46492994	
6	14000	14000	14000	14000	14000	14000	14000	14000		6	2.68E+08	2.32E+08	2.01E+08	1.69E+08	1.44E+08	1.19E+08	8.52E+07	62072007	
Cooling time										Fission Source									
(x,y)	1	2	3	4	5	6	7	8		(x,y)	1	2	3	4	5	6	7	8	
1	1	2	5	10	15	20	30	40		1	4.03E+07	4.68E+07	4.25E+07	3.66E+07	3.11E+07	2.57E+07	1.98E+07	12521188	
2	1	2	5	10	15	20	30	40		2	6.88E+07	8.12E+07	7.41E+07	6.34E+07	5.32E+07	4.33E+07	3.26E+07	20199639	
3	1	2	5	10	15	20	30	40		3	9.82E+07	1.17E+08	1.07E+08	9.08E+07	7.54E+07	6.05E+07	4.47E+07	27169878	
4	1	2	5	10	15	20	30	40		4	1.32E+08	1.58E+08	1.44E+08	1.21E+08	9.98E+07	7.93E+07	5.78E+07	34751134	
5	1	2	5	10	15	20	30	40		5	1.62E+08	1.92E+08	1.73E+08	1.45E+08	1.19E+08	9.42E+07	6.80E+07	40823941	
6	1	2	5	10	15	20	30	40		6	1.49E+08	1.74E+08	1.56E+08	1.30E+08	1.06E+08	8.38E+07	6.03E+07	36229288	
Response (experimental)										Response(Calculated)									
(x,y)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	(x,y)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5
0.5										0.5	0.123198	0.230998	0.221266	0.193583	0.166627	0.141523	0.114534	0.083892	0.036611
1.5		0.6				0.3				1.5	0.305453	0.580498	0.561644	0.491674	0.420538	0.353999	0.28285	0.203871	0.087599
2.5			0.8							2.5	0.467647	0.897903	0.880437	0.770597	0.653747	0.543993	0.427576	0.301569	0.127819
3.5				0.8						3.5	0.658686	1.271323	1.252983	1.094413	0.922518	0.761393	0.591298	0.410909	0.172554
4.5					1.4					4.5	0.879337	1.696988	1.669344	1.453015	1.219392	1.002015	0.772365	0.532245	0.222616
5.5						1.2				5.5	1.029258	1.978009	1.923356	1.665877	1.394836	1.14574	0.880125	0.605581	0.253192
6.5										6.5	0.57336	1.093457	1.058167	0.914234	0.765146	0.628852	0.482792	0.332139	0.139652
Response Difference										Response Difference									
(x,y)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	(x,y)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5
0.5										0.5									
1.5										1.5		3.36%						-15.25%	
2.5										2.5			3.82%						
3.5										3.5				-3.65%					
4.5										4.5									
5.5										5.5									4.74%
6.5										6.5									

Real-time simulations of commercial spent fuel pools

Criticality Safety, Nonproliferation & Safeguards applications



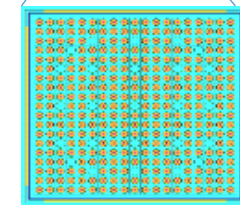
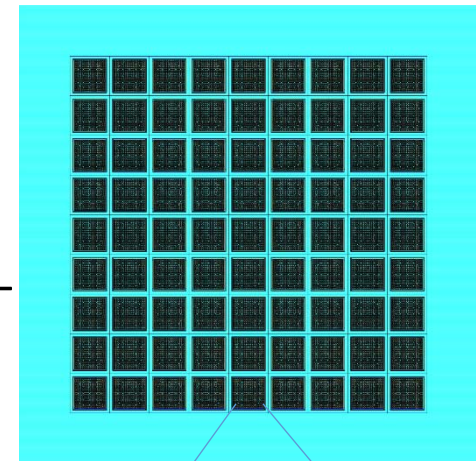
Background

- **Standard approach - Full Monte Carlo calculations face difficulties in this area**
 - Convergence is difficult due to low coupling between regions (due to absorbers)
 - Convergence can also be difficult to detect
 - Computation times are very long, especially to get detailed information
 - Changing pool configuration requires complete recalculation
- **Fission Matrix (FM) approach – It can address the above issues**
 - Fission matrix coefficients are pre-calculated using Monte Carlo
 - Computation times are much shorter, with no convergence issues
 - Detailed fission distributions are obtained at pin level
 - Changing pool assembly configuration does not require new pre-calculations (No additional Monte Carlo)

Developed a Multi-stage methodology for determination of FM coefficients

- As the computational size (for I²S reactor design)
 - $N = 9 \times 9 \times 336 = 27,216$ total fuel pins/ fission matrix cells
 - Considering 24 axial segments per rod, then
 - $N = 653,184$
- Standard FM would require $N = 653,184$ separate fixed-source calculations to determine the coefficient matrix
 - A matrix of size $N \times N = 4.26649E+11$ total coefficients (> 3.4 TB of memory is needed)
- The standard approach is clearly NOT feasible
- We have developed a multi-stage approach to obtain detailed FM coefficients (*in the process of filing for a patent*)

9x9 array of assemblies in a pool



Assembly with 19x19 lattice; 25 positions are reserved for control rods

RAPID tool

- **Developed the RAPID (Real-time Analysis spent fuel Pool *In situ* Detection) tool for determination of**
 - Eigenvalue
 - Subcritical multiplication
 - Pin-wise, axially-dependent fission density
- With application to
 - Criticality safety
 - Safeguards
 - Nonproliferation and materials accountability

RAPID code system - Structure

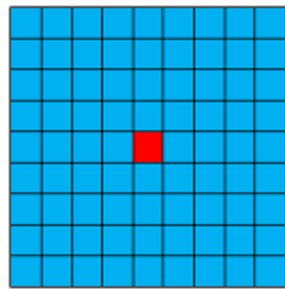
Pre-Calculation (one time):

1. Burnup Calculation – to obtain material composition
2. Fission Matrix Coefficient Generation

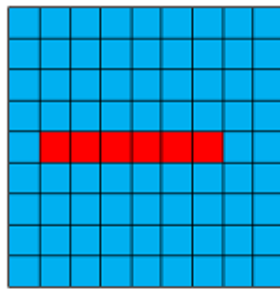
Real-time Analysis:

1. Run Fission Matrix Code
2. Process Results

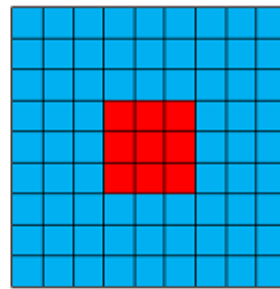
Test Problems (9x9 assemblies)



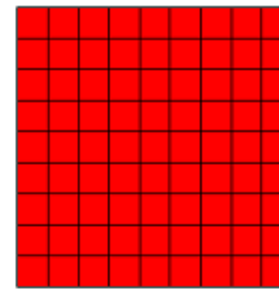
(a) Case 1



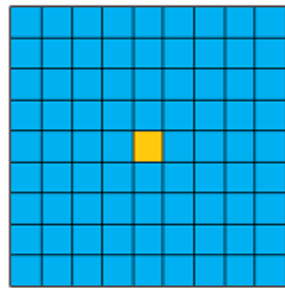
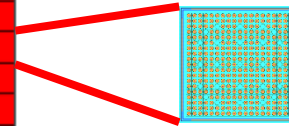
(b) Case 2



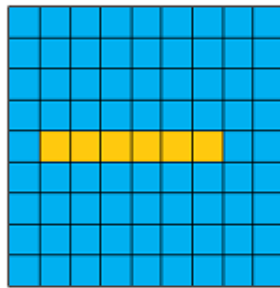
(c) Case 3



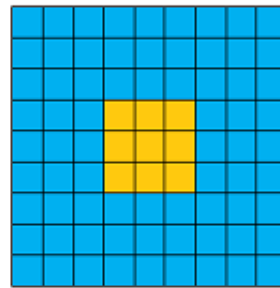
(d) Case 4



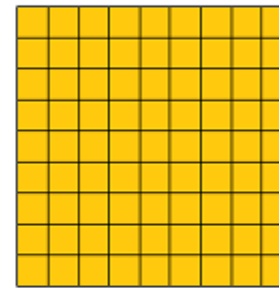
(e) Case 5



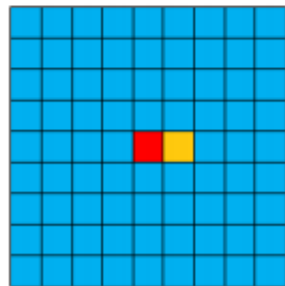
(f) Case 6



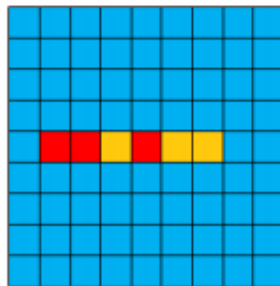
(g) Case 7



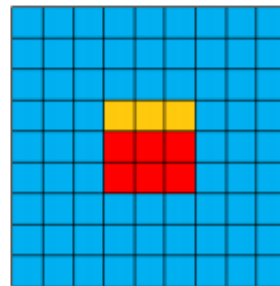
(h) Case 8



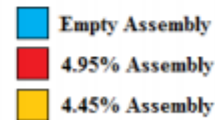
(i) Case 9



(j) Case 10



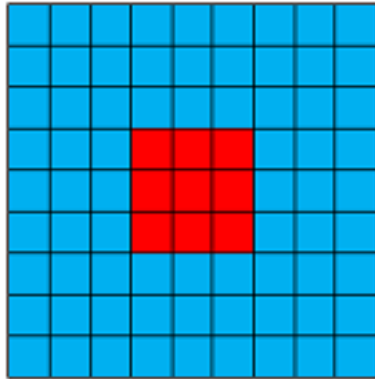
(k) Case 11



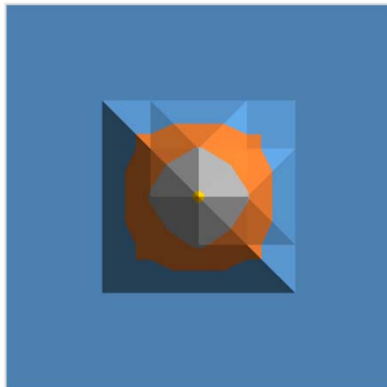
(l) Material Legend

Case #	Number of Assemblies	Fuel Type
1	1x1	4.95%
2	6x1	4.95%
3	3x3	4.95%
4	9x9	4.95%
5	1x1	4.45%
6	6x1	4.45%
7	3x3	4.45%
8	9x9	4.45%
9	2x1	Mixed
10	6x1	Mixed
11	3x3	Mixed

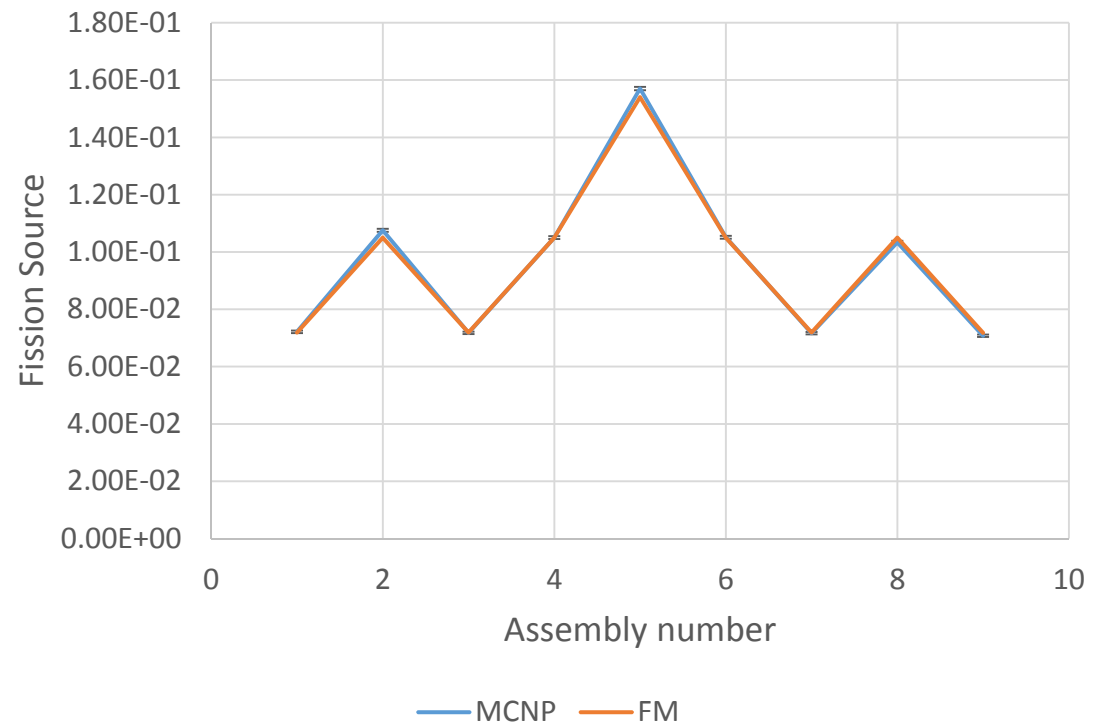
Case 3 Eigenfunction



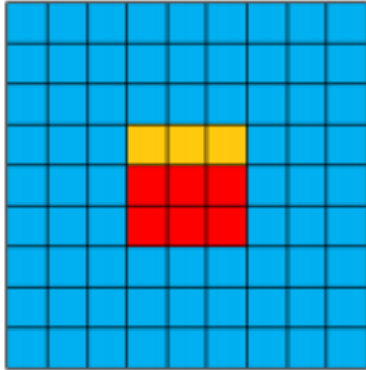
Reference Solution



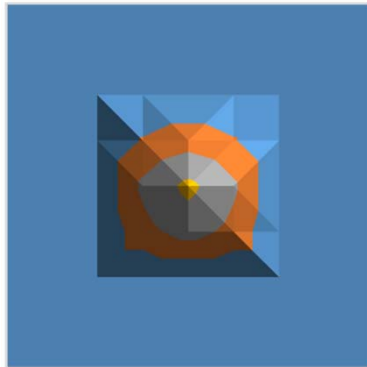
Comparison of RAPID with MC



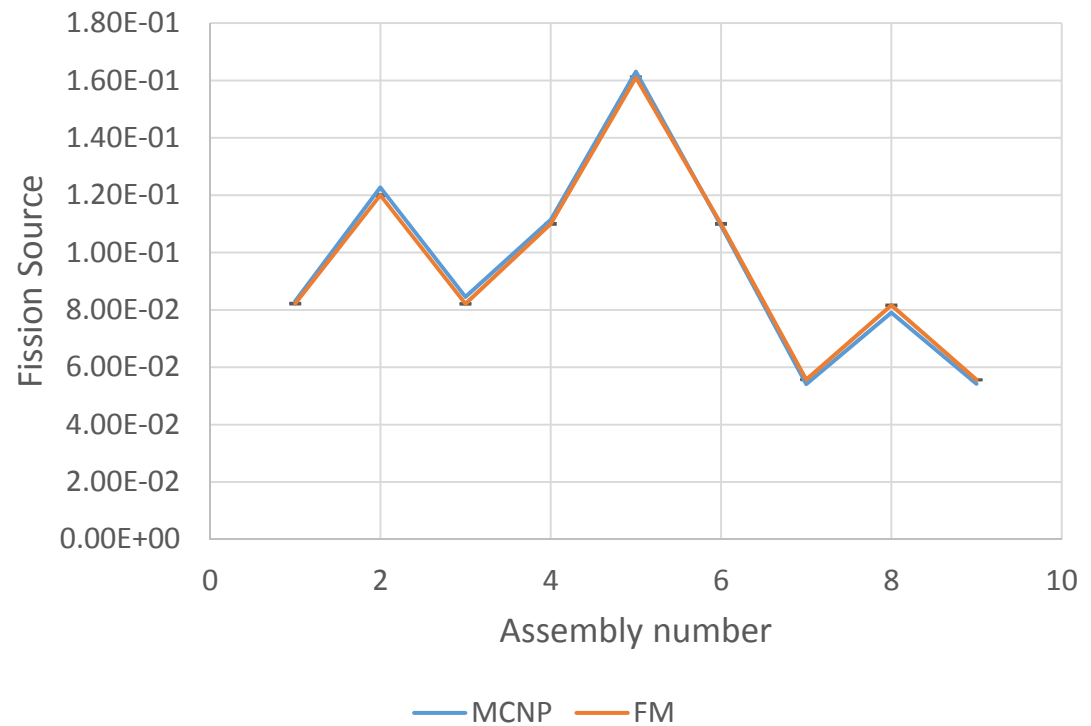
Case 11 Eigenfunction



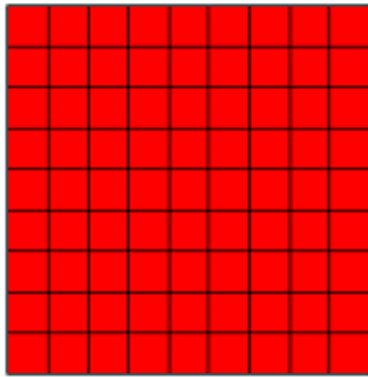
Reference Solution



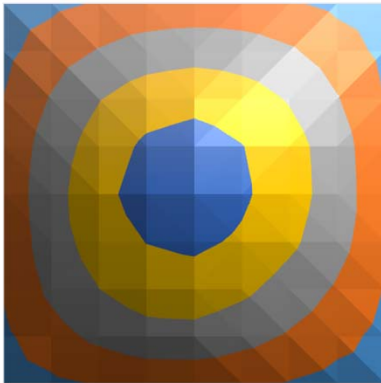
Comparison with RAPID with MC



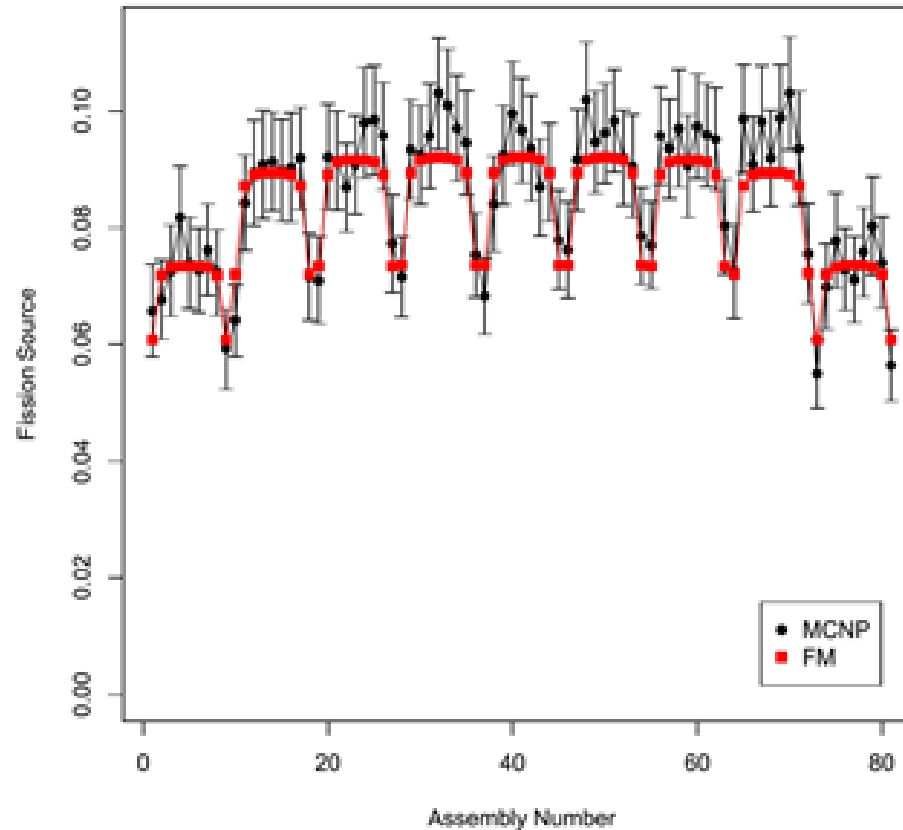
Case 4 Eigenfunction distribution



Reference Solution



Comparison with RAPID with MC



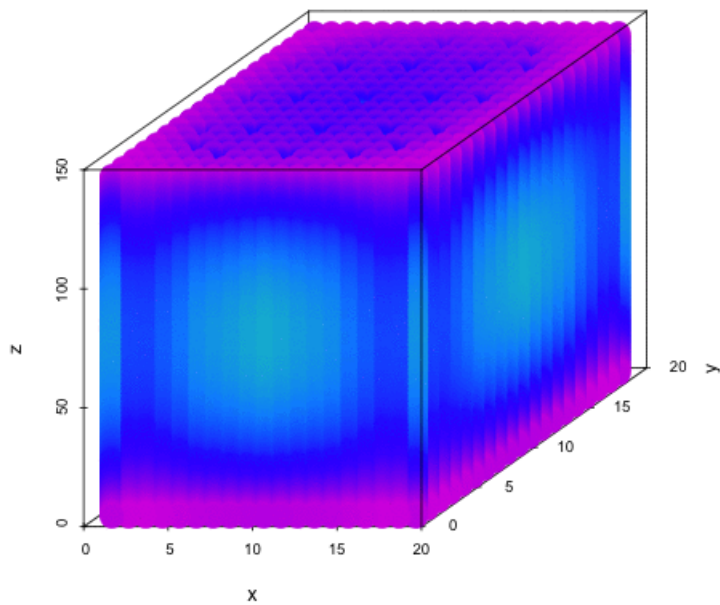
Comparison of calculated M - RAPID vs. MCNP

Case	FM		MCNP			Error in M (FM vs MCNP)	Speedup (FM vs MCNP)*
	M	Time (min)	M	Time (min)	1- σ Uncertainty		
1x1	3.343353	0.092	3.33155	925	0.0010	0.35%	10062
6x1	4.328244	0.213	4.31336	1198	0.0010	0.35%	5613
3x3	5.428051	0.965	5.40992	1502	0.0011	0.35%	1558
9x9	6.697940	8.17	6.67674	1928	0.012	0.32%	236

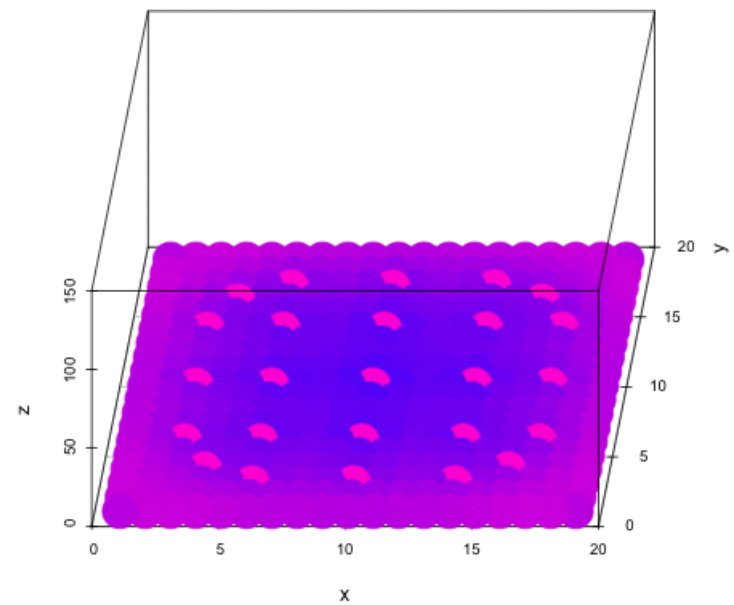
*Note that the *RAPID* also provide **pin-wise, axial-dependent fission source** or power.

3-D Fission Density

Y-LEVEL ANIMATION



Z-LEVEL ANIMATION



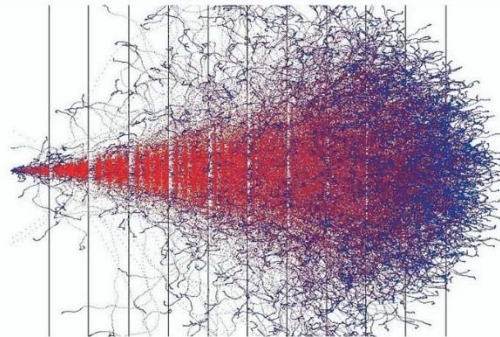
Conclusion

MRT methodology allows for development of real-time tools for analysis of nuclear systems

Thanks!

Questions?

Monte Carlo Methods for Particle Transport



Alireza Haghghat

 CRC Press
Taylor & Francis Group